Gender and Racial Peer Effects with Endogenous Network Formation*

Chih-Sheng Hsieh†
Department of Economics
Chinese University of Hong Kong

Xu Lin‡
Department of Economics
Wayne State University

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Abstract

We apply a high order spatial autoregressive (SAR) model to simultaneously capture heterogeneous peer effects from multiple gender and racial groups, and endogenous network formation. In students' GPA and smoking behaviors, we find that within-gender endogenous effects are stronger than across gender. Females and whites are more sensitive to peer influences and more influential than other students. Intra-race spillover effects are stronger than inter-race effects for whites, but not for non-whites. For contextual effects, we show that peers' age, race and family background, but not gender composition, are relevant for GPA and smoking behaviors. Homophily in observed and unobserved characteristics are important for friendship formation.

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†Department of Economics, The Chinese University of Hong Kong, 9/F, Esther Lee Building, CUHK, Shatin. Tel: 3943-8006. Fax: 2603-5805. Email: cshsieh@cuhk.edu.hk.

‡Department of Economics, Wayne State University, 656 W. Kirby, Detroit MI 48202. Tel: 313-577-2725. Fax: 313-577-9564. Email: xulin@wayne.edu.

1 Introduction

Peer influences have been investigated intensively for a variety of behaviors and outcomes.[1] However, many existing studies focus only on the average magnitude of peer influences, without paying special attention to the possible heterogeneity nature along the gender and race dimensions of peer effects. As a result, informative policy implications are difficult to draw from those studies. For instance, to shed light on the debate about the benefits of single-sex versus coeducational schooling, we need to know the mechanism of peer effects along the gender dimension, i.e., how an individual affects and is affected by his/her male and female peers differently. Another example is the debate on school segregation versus desegregation, which calls for the analysis on the mechanism of peer interactions within and across racial groups.

Recently, a few studies have started to explore the heterogeneity of peer effects along the lines of gender and race. However, most of these studies focus on the effects of gender or racial composition, in particular, female or black proportion, in a group on various outcomes.[2] These effects are termed "contextual effects" or "exogenous effects" by Manski (1993), which represent the effects of peers' predetermined characteristics, gender or race in this case, on an individual's outcome. Another type of peer effects are "endogenous effects", which capture the effects of peers' contemptuous outcomes on an individual's outcome, and have only been examined in a few recent papers.[3] A typical assumption made in these studies is the absence of contextual peer effects.[4] As pointed out by Bramoullé *et al.* (2009), Fruehwirth (2010, 2013), Lee *et al.* (2010), Lin (2010),

[1] For example, for students' academic achievement (Hanushek *et al.* 2003; Hoxby 2000; Lin 2010; Sacerdote 2001; Zimmerman 2003); students' use of tobacco, alcohol, and substance (Clark and Lohéac 2007; Fletcher 2010 and 2011; Gaviria and Raphael 2001; Lundborg 2006; Powell *et al.* 2005); school enrollment decisions (Bobonis and Finan 2009; Lalive and Cattaneo 2009); the spreading of obesity (Fortin and Yazbeck 2011); participation of retirement plan (Duflo and Saez 2002), and so on.

[2] See Hoxby (2000), Lavy and Schlosser (2011), and Whitmore (2005) on gender proportion, and Angrist and Lang (2004), Hoxby (2000), and Hanushek *et al.* 2009 on racial composition.

[3] Clark and Lohéac (2007), Kooreman (2007), Soetevent and Kooreman (2007) and Trogdon *et al.* (2008) study endogenous peer effects along the dimension of gender, Nakajima (2007) investigates both gender and racial endogenous interaction effects.

[4] A notable exception is Fruehwirth (2013), who examines not only the usual racial composition effects, but also the contemporaneous peer achievement spillovers of different racial groups. But her model specification and identification are different from ours, as will be discussed in Section 2.

among others, separately identifying contextual and endogenous effects is important as endogenous effects generate a social multiplier effect whereas contextual effects do not. Therefore, in order to provide sound policy implications regarding single-sex versus coeducational schooling, as well as school segregation versus desegregation, it is necessary to identify not only the contextual effects along the dimensions of gender and race, but also the endogenous effects generated by peers of different gender and racial groups.

Focusing on only one type of the peer effects in most existing studies is due in great part to the identification challenge in peer effects estimation, i.e., the "reflection problem" (Manski 1993), which refers to the impossibility to separate endogenous effects from contextual effects in the conventional linear-in-means model without imposing priori restrictions on the parameters. To circumvent the "reflection problem", conventional studies include only one type of social effects in the model. Fortunately, recent developments in spatial autoregressive (SAR) models demonstrate that the "reflection problem" is not an issue for SAR models as the nonlinearity introduced in the peer measurement breaks down the linearity between endogenous and contextual peer measures (e.g., Bramoullé *et al.* 2009; Calvó-Armengol *et al.* 2009; Lee *et al.* 2010; Lin 2010). In this study, we employ a generalized SAR model to identify not only the effects of gender and racial composition on students' academic achievement and smoking behaviors, but also the contemporaneous spillover effects from the outcomes of peers from different gender and racial groups, an under-explored feature in the literature. We investigate, for instance, whether the effects of black students to white students are the same as those from whites to blacks, given some empirical evidences that blacks smoke much less than whites, as well as the substantial achievement gap between blacks and whites (e.g., Echenique *et al.* 2006; Kandel *et al.* 2004).

Another identification difficulty in peer effects estimation is the omitted variable bias problem, caused by either endogenous group formation or unmeasured variables, e.g., school resources, teacher quality, etc. (Moffitt 2001). To partially address this issue, existing studies exploit either instrumental variable (IV) strategy (e.g., Evans *et al.* 1992; Rivkin 2001); or the experiment nature of the data (e.g., Sacerdote 2001; Zimmerman 2003); or group fixed effect strategy (e.g., Bramoullé *et al.* 2009; Calvó-Armengol *et al.* 2009; Lin 2010). However, for the IV strategy, it is not easy to find a valid IV which is correlated with the peer variables while uncorrelated with the structural errors. For the experiment type strategy, the requirements on data are rather strict. And the fixed effect strategy can capture the unobservables fixed at the group level, but not those varying within the group.

As pointed out in Lin (2010) and Nathan (2008), among others, a more sophisticated strategy to deal with the omitted variable bias problem is to develop an equation system to simultaneously model both endogenous peer network formation and peer influences, which has recently been investigated by Hsieh and Lee (2013), Goldsmith-Pinkham and Imbens (2013), and among others. In particular, Hsieh and Lee (2013) propose a network formation model in which distances of unobserved variables are used to capture the influence of homophily on omitted variables in forming friendship links. These unobserved variables are then applied to the SAR model as the control function to correct for the bias caused by omitted variables. However, Hsieh and Lee (2013) only consider peer interactions within one homogenous group and therefore the obtained peer effect can be best explained as the average peer effect.

In this study, we use a high order SAR model to incorporate heterogeneous peer effects from peers of multiple gender and racial groups while accounting for endogenous network formation using the modeling approach of Hsieh and Lee (2013). Along the line of gender, we include four spatial weighting matrices to capture the contemporaneous spillover effects from female to female, male to female, male to male, female to male, respectively. For cross-race analysis, we specify nine weighting matrices to capture the behavior spillover effects from white to white, black to white, other racial groups to white, black to black, white to black, and the like.[5] Estimating the high order SAR model would be more complicated than the single order SAR model as the Jacobian determinant in the log likelihood function of the high order SAR model cannot be easily calculated (Lee and Liu 2010). Besides, a constrained optimization for multidimensional endogenous effect parameters to satisfy the stability condition needs to be performed. However, LeSage and Pace (2009) indicate that the Bayesian approach with the Markov Chain Monte Carlo (MCMC) sampling may have an advantage in estimating the high order SAR model because the stability restriction can be easily imposed during the rejection step of the Metropolis-Hastings algorithm. In this paper we thus follow the Bayesian estimation approach.

Data are from the National Longitudinal Study of Adolescent Health (Add Health). Our results indicate that failing to include both endogenous and contextual social effects in the model, or

[5]We divide race into three categories to analyze racial peer effects: white, black and other racial groups.

to control for the endogenous group formation could seriously bias the estimated peer effects, especially for students' academic achievement (GPA). For gender endogenous spillover effects, we find that within-gender interaction effects are stronger than cross-gender effects, and females are subject to larger within-gender spillover effects and more influential across gender. In terms of racial spillover effects, our findings indicate that whites are most sensitive to contemporaneous peer influences and most influential compared to black students and students from other racial groups. Furthermore, for whites, intra-race spillover effects are stronger than inter-race effects. While for blacks and students of other racial groups, the inter-race social interaction effects generated by their white friends appear to be slightly stronger than intra-race interaction effects. And blacks appear to be the least influential group across race, as they do not generate any significant endogenous effect on their non-black friends. An important implication is that policy intervention targeting at female and/or white students will be most effective due to the influential status of female and/or white students as well as their responsiveness to peer influences. For contextual effects, we find that peers' age, race and family background such as mother's education and mother's occupation, could generate significant effect on GPA and smoking behaviors. In contrast, gender composition of peers do not appear to matter. The parameters from the network formation model show that homophily in terms of both observed factors such as grade, race and gender, as well as unobserved characteristics are important for friendship formation.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 discusses our model in details. Section 4 describes the data and empirical results are presented in Section 5. Section 6 briefly concludes. Technical details are provided in the Appendix.

2 Literature Review

This section reviews some papers that examine social interactions along the lines of gender and race. As mentioned above, most existing studies focus on the contextual effects of female or black proportion in a group. Some studies also estimate separated regressions for male and female or for different racial subgroups to investigate how male and female or people of different races respond differently to gender or racial composition in the group. Studies regarding endogenous gender or racial peer effects are rather limited.

Hoxby (2000) estimates gender and racial peer effects in Texas elementary schools by exploiting exogenous idiosyncratic variations in gender and race composition in a school-grade in adjacent years. Regarding gender composition effects, she finds that both male and female achieve higher test scores in reading and math when there are more female in the class. Using data from an introductory undergraduate management course, Hansen *et al.* (2006) find that male-dominated groups tend to perform worse than female-dominated and equally mixed gender groups in several outcomes. Relying on within school variations in the proportion of female students across adjacent cohorts in Israeli schools, Lavy and Schlosser (2011) show that a higher proportion of female peers has a positive and significant effect on academic achievements, with similar magnitude for both genders. They also find a positive and significant effect of female share on high school math and science classes enrollment rates for boys, while a smaller and less significant effect for girls. Han and Li (2009) use a randomized experiment data from a Chinese college and find that only female students respond to their roommates' influences. On the other hand, Whitmore (2005) exploits a randomized experiment data from Tennessee's Project STAR, and finds mixed results for the effects of female proportion. Christofides *et al.* (2011) show that having controlled for other more precise peer variables and individual characteristics, the effect of female proportion in high school is insignificant for either gender. Regarding endogenous gender peer effects, Soetevent and Kooreman (2007) find that within-gender effects are significantly stronger than cross-gender interactions for student truancy, after controlling for school fixed effects. Using a similar school fixed effect strategy, Clark and Lohéac (2007) find that young males are more influential than young females, but both young men and women are just as influenceable for several risky behaviors. Relying on both school fixed effects and instrument variable strategies, Trogdon *et al.* (2008) show that females are more responsive to peer BMI and overweight status than males, but no evidence that intra-gender effects are stronger. Using the same technique as Soetevent and Kooreman (2007), Kooreman (2007) finds that within-gender interactions are larger than cross-gender interactions for several outcomes, and that the effect of girls on boys is generally larger than that of boys on girls. But cross-gender interactions are found to be stronger than within-gender interactions for alcohol expenditures. The results in Nakajima (2007) indicate that peer interactions are stronger within than across genders on students' smoking behaviors.

A few studies have examined racial group based peer effects. The results in Hoxby (2000) show that intra-race effects are stronger than inter-race effects and that all students perform worse with a large black share in the group. Using an approach similar to Hoxby (2000), Hanushek *et al.* (2009) study the effects of racial shares in school on students' academic achievement and find that a higher percentage of black schoolmates negatively affects mathematics achievement of blacks, while the effect on whites is insignificant. Angrist and Lang (2004) investigate the impacts of the Metco — Massachusetts voluntary interdistrict integration program — on students in the receiving districts. They find that blacks in the receiving district are more affected by the influx of lower-achieving blacks brought by the Metco program, while white students in the receiving district are unaffected. Several other studies, including Boozer, Krueger and Wolkon (1992), Grogger (1996), Hanushek and Raymond (2005), and Nathan (2008), also find evidence for the effects of school racial composition on academic and other outcomes. Meanwhile, Rivkin (2000) shows that exposure to whites does not appear to increase academic attainment or earnings for blacks. Cook and Evans (2000) find no evidence that racial concentration affects the black-white difference in National Assessment of Educational Progress scores. With regard to endogenous racial peer effects, Nakajima (2007) finds that within-race peer effects are generally positive and significant, and that peer effects between white and black students are insignificant in smoking decisions, after controlling for county fixed effects.

To the best of our knowledge, the only study that explores both contextual effects of racial composition and contemporaneous spillovers from peers of different races is Fruehwirth (2013). She examines classroom racial effects in North Carolina public elementary school students by generalizing the conventional linear-in-means model to incorporate the contemporaneous achievement spillovers from the classmates, i.e., the average achievements of white and non-white classmates, as well as the racial composition, i.e., the proportions of white and non-white students in the class. The identification is achieved by an exclusion restriction generated by a student accountability policy, which is similar to the difference in difference strategy.[6] Her results indicate a lack of cross-racial behavioral spillovers: white students receive positive achievement spillovers only from their white classmates and non-white students receive positive spillovers from their non-white peers only. For contextual peer effects, she find that a higher proportion of non-white classmates

[6]As mentioned in that paper, "peer effects are identified by comparing classrooms with similar compositions of low-achievers (those potentially affected by the policy) pre- and post-accountability."

negatively affects the performance of both whites and no-whites. However, the author does not explicitly study the effects of and on the blacks, neither does she study peer effects along the gender dimension. In addition, due to data availability, peers are defined at the classroom level, instead of the more relevant level of friendship networks.[7] More importantly, the possible endogenous formation of peer groups is not fully modeled.

3 Our Approach

In this paper we employ a high order SAR model with both endogenous and contextual effects, as well as endogenous network group formation, to study heterogenous peer effects along the lines of gender and race. Compared to previous applications of SAR models in Lee *et al.* (2010), Lin (2010), Hsieh and Lee (2013), and others, where only a homogenous endogenous effect is studied, we consider a specification of multiple spatial weighting matrices in the SAR model to capture influences from friends with different characteristics and therefore turn the single order SAR model to a high order one. Furthermore, our structural model system captures endogenous network formation and peer influences simultaneously.

3.1 The High Order SAR Model

We consider an environment where individuals are placed in groups such as schools, $g = 1, \cdots, G$. Two types of outcome variables of individual i in group g are observed: one is a continuous variable $y_{i,cg}$, such as a student's academic performance measured by GPA and the other is a Tobit-type variable $y_{i,tg}$ with left censoring at zero, such as a student's smoking frequency. The m_g-dimensional vectors $Y_{cg} = (y_{1,cg}, y_{2,cg}, \cdots, y_{m_g,cg})'$ and $Y_{tg} = (y_{1,tg}, y_{2,tg}, \cdots, y_{m_g,tg})'$ summarize the outcome variables of m_g individuals in group g. Let $x_{i,g}$ be a k-dimensional row vector containing individual i's exogenous characteristics and the $m_g \times k$ dimensional matrix X_g be a collection of such vectors in group g. The friendship network of individuals in group g is represented by a $m_g \times m_g$ spatial weighting matrix (adjacency matrix; sociomatrix) W_g. Each entry of W_g, $w_{ij,g}$, is a dichotomous indicator which equals one if individual i sends a friendship nomination to individual j and

[7] As demonstrated in many studies, such as Bramoullé *et al.* (2009), Calvó-Armengol *et al.* (2009), Haynie (2001, 2002), and Lin (2010), adolescents are significantly influenced by their friends in a variety of outcomes.

zero, otherwise. The nomination is directed without guaranteed reciprocality.

To distinguish heterogeneous peer effects, we divide friends of different characteristics, e.g., gender and race, into several subgroups. For example, friends of an individual can be divided into two subgroups based on gender – one for female friends and the other for male friends. Under the SAR model setting, given $\bar{p}$ subgroups in group g, the spatial weighting matrix W_g can be divided into $\bar{p}\times\bar{p}$ blocks, $\{W_g^{pq}\}_{p,q=1}^{\bar{p}} = (W_g^{11}, W_g^{12}, \cdots, W_g^{\bar{p}\bar{p}})$, where W_g^{pq} is the matrix of links between subgroups p and q in group g.[8] By assigning different coefficients to represent different peer influences within and across blocks, the SAR model can be specified as

$$Y_{cg} = \widetilde{W}_g(\lambda_c)Y_{cg} + X_g\beta_{1c} + \widetilde{W}_g X_g \beta_{2c} + l_g\alpha_{cg} + \varepsilon_{cg}, \quad \varepsilon_{cg} \sim i.i.d.\mathscr{N}_{m_g}(0, \sigma^2_{\varepsilon_c} I_{m_g}), \quad g = 1, \cdots, G, \tag{1}$$

where

$$\widetilde{W}_g(\lambda_c) = \begin{pmatrix} \lambda_{11,c}\widetilde{W}_g^{11} & \cdots & \lambda_{1\bar{p},c}\widetilde{W}_g^{1\bar{p}} \\ \vdots & \ddots & \vdots \\ \lambda_{\bar{p}1,c}\widetilde{W}_g^{\bar{p}1} & \cdots & \lambda_{\bar{p}\bar{p},c}\widetilde{W}_g^{\bar{p}\bar{p}} \end{pmatrix}$$

with $\widetilde{W}_g^{pq}$ represents the normalized W_g^{pq} by the row-sum; l_g is the m_g-dimensional vector of ones; $\mathscr{N}_{m_g}$ represents a multivariate normal distribution of dimension m_g and I_{m_g} is the identity matrix of dimension m_g.

In Eq. (1), λ_{pp}, $p = 1, \cdots, \bar{p}$, on the diagonal of $\widetilde{W}_g(\lambda_c)$ represent peer effects within the same subgroups and other λ_{pq}, $p \neq q$ represent peer effects across subgroups. Alternatively, Eq. (1) can be rewritten as:

$$Y_{cg} = \lambda_{11,c}\widetilde{W}_{11,g}Y_{cg} + \cdots + \lambda_{\bar{p}\bar{p},c}\widetilde{W}_{\bar{p}\bar{p},g}Y_{cg} + X_g\beta_{1c} + \widetilde{W}_g X_g\beta_{2c} + l_g\alpha_{cg} + \varepsilon_{cg}, \quad g = 1, \cdots, G, \tag{2}$$

where $\widetilde{W}_{pq,g}$ is a $m_g \times m_g$ matrix with the corresponding $(p,q)^{\text{th}}$ block equal to W_g^{pq} and 0 elsewhere. The terms $\{\widetilde{W}_{pq,g}Y_{cg}\}_{p,q=1}^{\bar{p}}$ capture the contemporaneous outcomes of peers and the coefficients $\{\lambda_{pq,c}\}_{p,q=1}^{\bar{p}}$ represent the heterogeneous peer (endogenous) effects. The terms X_g and $\widetilde{W}_g X_g$ and their coefficients capture the own and contextual effects from exogenous characteristics.

[8] Before we can divide the spatial weighting matrix W_g into $\bar{p}\times\bar{p}$ blocks, we should first reorder individual observations in Y_g, X_g, as well as W_g based on individual's characteristic.

We can incorporate heterogenous contextual effects in Eq. (2) similarly.[9] The term α_{cg} represents the group fixed effect, which captures the effects of the environmental unobservables (correlated effects) shared by all individuals in the same group, e.g., teacher quality, campus facility, etc. in a school setting.

For Tobit-type outcome variables Y_{tg}, we can specify a high order SAR model for the vector of latent outcomes $\ddot{Y}_{tg}$ as follows:

$$
\begin{aligned}
Y_{tg}(W_g) &= \max\left(0, \ddot{Y}_{tg}\right) \\
\ddot{Y}_{tg} &= \lambda_{11,t}\widetilde{W}_{11,g}Y_{tg} + \cdots + \lambda_{\bar{p}\bar{p},t}\widetilde{W}_{\bar{p}\bar{p},g}Y_{tg} + X_g\beta_{1t} + \widetilde{W}_g X_g \beta_{2t} + l_g\alpha_{tg} + \varepsilon_{tg}, \quad g = 1, \cdots, G.
\end{aligned} \tag{3}
$$

We use observed outcomes Y_{tg} instead of latent outcomes $\ddot{Y}_{tg}$ on the right hand side of Eq. (3) to capture peer effects, which implies that individuals only receive peer influences from their friends with $y_{j,tg} > 0$ but not from their friends with $y_{j,tg} = 0$.

The high order SAR models (without endogenous peer group formation) have been specified in several studies and estimated by the maximum likelihood (ML) method (Blommestein 1983, 1985), the general two-stage least square (G2SLS) method (Anselin and Bera 1998), or the general method of moments (GMM) approach (Lee and Liu 2010). In this paper we demonstrate that the Bayesian method has unique advantage over the traditional methods in estimating the high order SAR model, especially when endogenous network group formation is taken into account.

3.2 Endogenous Network Formation

The advantage of using SAR models for studying social interactions is that one can prevent the "reflection problem" inherited in the linear-in-means models. The identification of endogenous effects in the SAR model comes from the fact that friends of individuals are not perfectly overlapped. Hence, variations on exogenous characteristics of friends' friends will be valid instruments for endogenous effects. However, the concern of the SAR model in studying interactions is the possible endogenous friendship formation. If friendship formation and outcome variables are influenced by common unobserved factors which are not controlled for in the model, the estimates of peer effects

[9] However, to better focus on endogenous effect, as well as to keep the model parsimonious, we do not consider heterogenous contextual effects in this paper.

could be biased. For instance, students' ability level may affect not only their learning outcomes, but also their friendship choices, i.e., high ability students may choose to associate with other high ability students. The estimated peer effects on academic outcomes would be upward biased if ability is not taken into account.

To deal with the endogeneity problem, Hsieh and Lee (2013) propose a structural approach to simultaneously model the network formation and social interactions. They specify latent variables in both network formation and selection-corrected SAR (SCSAR) models to capture unobserved factors such as ability, taste, and others which may affect both friendship formation and outcomes. As the number of underlying unobserved factors is unknown, Hsieh and Lee (2013) specify latent variables in multiple dimensions. In both of their simulation and empirical studies, they show that once enough dimensions of latent variables are controlled for, the endogeneity bias on the estimated peer effect caused by friendship selection can be corrected.[10] In this paper, we extend the single order SCSAR model to a higher order one:

$$Y_{cg} = \lambda_{11,c}\widetilde{W}_{11,g}Y_{cg} + \cdots + \lambda_{\bar{p}\bar{p},c}\widetilde{W}_{\bar{p}\bar{p},g}Y_{cg} + X_g\beta_{1c} + \widetilde{W}_g X_g \beta_{2c} + Z_g\rho + l_g\alpha_{cg} + u_{cg}, \tag{4}$$

where $Z_g = (z_{1,g}, \cdots, z_{m_g,g})'$ represents unobservables which are correlated with W_g and $u_{cg} = \varepsilon_{cg} - Z_g\rho$ is assumed from i.i.d. $\mathcal{N}_{m_g}(0, \sigma_u^2 I_{m_g})$ for $g = 1, \cdots, G$. The term $Z_g\rho$ in Eq. (4) serves as a control function to deal with the endogeneity of W_g.[11] All regressors in Eq. (4) are uncorrelated with u_{cg} and thus the problem of endogenous bias dissolve.

To explicitly model the relationship between Z_g and W_g, Hsieh and Lee (2013) consider a network formation model, where the distances of individual unobservables $z_{i,g}$'s (in a dimension $\bar{d}$) are used to explain homophily of unobserables on each friendship link decision. This network

[10] In an independent study done by Goldsmith-Pinkham and Imbens (2013), they also propose to use the latent variable to control unobserved individual heterogeneity in both the network formation and interaction models. However, the latent variable considered in their approach is binary and only has one dimension. Maybe due to this impractical specification, they did not find any evidence of bias correction resulted from their approach in the empirical application.

[11] See discussions of the control function approach in Navarro (2008) and Wooldridge (2010).

formation model is specified as:

$$P(w_{ij,g}|c_{ij,g},z_{i,g},z_{j,g}) = \frac{\exp(\psi_{ij,g}w_{ij,g})}{1+\exp(\psi_{ij,g})},$$
$$\psi_{ij,g} = c_{ij,g}\gamma + \delta_1|z_{i1,g}-z_{j1,g}| + \cdots + \delta_d|z_{i\bar{d},g}-z_{j\bar{d},g}|. \tag{5}$$

In Eq. (5), $c_{ij,g}$ represents a $\bar{q}\times 1$ vector of observed dyad-specific variables, which captures the distances of observed characteristics, such as age, gender and race, between individuals i and j. By conditioning on observed variables $C_g = \{c_{ij,g}|i,j\in \text{group } g\}$ and unobserved variables Z_g, each network link decision is assumed to be independent and thus for the whole network, we have:

$$P(W_g|C_g,Z_g) = \prod_{i}^{m_g}\prod_{j\neq i}^{m_g} P(w_{ij,g}|c_{ij,g},z_{i,g},z_{j,g}). \tag{6}$$

The high order SCSAR model in Eq. (4) and the network model in Eqs. (5) and (6) form a system to study social interaction effects, which captures the possible selection in friendship formation and changes in outcomes due to unobservables. The joint probability of Y_{cg} and W_g is

$$P(Y_{cg},W_g|X_g,C_g,\theta_c,\alpha_{cg}) = \int_{Z_g} P(Y_{cg}|W_g,X_g,Z_g,\theta_c,\alpha_{cg})\cdot P(W_g|C_g,Z_g,\theta_c)\cdot f(Z_g)dZ_g, \tag{7}$$

where $\theta_c = (\gamma',\delta',\lambda_c',\beta_{1c}',\beta_{2c}',\sigma_u^2,\rho')$.[12]

As mentioned, the way the SCSAR model corrects for the endogeneity problem is in line with the control function approach, or the Heckman (1979) selection model in some aspects. It is known that the identification of the Heckman selection model relies on the joint normality between error terms and the exclusion restriction on exogenous variables, therefore, we would like to discuss the role of these two identification conditions in our system of network formation and SCSAR models. First, in order to decompose ε_{cg} in Eq.(2) into $Z_g\rho + u_{cg}$ in Eq. (4), we may either assume (i) there is a joint normality between latent variables $z_{i,g}$ and the error term $\varepsilon_{i,cg}$, or (ii) the conditional expectation of $\varepsilon_{i,cg}$ given $z_{i,g}$ is linear, and $z_{i,g}$ is from a known distribution. The second assumption is weaker and suggested by Olsen (1980). Both assumptions feature the use of parametric estimation approach based on full information likelihood. In this paper we adopt the second assumption and assume the distribution of $z_{i,g}$ is normal. Second, although we use the full information likelihood to estimate the SCSAR model, which is different from the inverse Mills ratio used by Heckman,[13]

[12] Replacing Y_{cg} by Y_{tg} in Eq. (7), we can get the probability specification for Tobit-type variables.

[13] The inverse Mills ratio is quasi-linear over a wide range of its arguments. Therefore, the set excluded variables (excluded from the main equation) is necessary for achieving identification of the inverse Mills ratio.

the exclusion restriction would still help us to identify the model. It worths noticing that in our system there are natural exclusion restrictions (Some exogenous variables used in the network formation model are dyad-specific, which are naturally excluded from the SCSAR model). To provide more details about the identification of parameters in the model, we demonstrate a semiparametric identification strategy in Appendix A.

3.3 Bayesian Estimation

We estimate the high order SCSAR and the network models by the Bayesian method. The effectiveness of the Bayesian method in handling estimation of models with random effects or latent variables is demonstrated in Zeger and Karim (1991), Chib and Carlin (1999), and others. The procedure of Bayesian estimation is to specify first the prior distributions of the unknown parameters and the unobservables in the models, such as unobserved characteristics $\{Z_g\}$, and groups fixed effects $\{\alpha_g\}$, according to the prior information. Then, given the prior distributions and the likelihood functions of the models, we can derive the posterior distributions of the parameters and use the MCMC sampling to simulate random draws from them to obtain the parameter estimates. Note that during the posterior simulation, the unobservables of the models, such as unobserved characteristics $\{Z_g\}_{g=1}^{G}$, groups fixed effects $\{\alpha_g\}_{g=1}^{G}$, and latent variables $\{\ddot{Y}_{tg}\}_{g=1}^{G}$ for the Tobit case, are simulated from their conditional posterior distributions and are used as augments in the conditional posterior distributions of the model parameters to simplify or facilitate their sampling (Albert and Chib 1993, Handcock *et al.* 2007). Details about the prior specifications and the derivation of the posterior distributions are provided in Appendix B.

One question remains unanswered is how to choose the dimension of unobservables in the network formation as well as SCSAR models. The optimal dimension should be determined during the estimation procedure by appropriate model assessment tools. The standard model assessment tool under the Bayesian framework is the Bayes factor, which is the ratio of marginal likelihoods of two competing models. In practice, researchers usually use the MCMC methods to simulate the marginal likelihood of the model (Chib 1995, and Chib and Jeliazkov 2001). However, when random effects or latent variables are involved in the models like our SCSAR and network models, using the MCMC method to calculate the model marginal likelihood would be difficult. Therefore, we evaluate SCSAR models with unobservables of different dimensions using the measure of AICM (Akaike's information criterion - Monte Carlo) proposed by Raftery *et al.* (2007). AICM is an estimate of the conventional AIC, which is not directly obtained from the posterior simulation as the maximum loglikelihood value may not be available. Given an appropriate assumption on the distribution of the loglikelihoods from each posterior draws, we can obtain an estimate of AIC as well as its standard error. Similar to the case of conventional AIC, the model with the highest AICM value is the desired model. The assumptions on the distribution of the loglikelihoods and derivation of the AICM are provided in Appendix C.

4 Data Summary

We employ data from Add Health, a nationally representative survey of adolescents enrolled in grades 7-12 from 132 schools. Add Health provides information regarding respondents' demographic backgrounds, academic performances, health related behaviors, as well as the most valuable information on their friendship networks. Four waves of surveys were conducted from 1994 to 2008. In the Wave I in-school survey, a total 90,118 students were interviewed and each respondent was asked to nominate up to five male and five female friends. We use such information to construct students' friendship networks.

From 132 schools, we pick 14 high schools with students in in grades 9-12 as our sample.[14] Each school is a network group in our analysis. There are 2,841 (3,369) respondents in the sample of GPA (smoking).[15] The dependent variables of GPA are constructed as the average grades of 4 subjects: English/Language Arts, Mathematics, History/Social Studies and Science. The dependent variables of smoking are obtained from the survey questions: "*During the past twelve months, how often did you smoke cigarettes*?". We transform the responses of the students to a monthly basis to obtain the Tobit variable of smoking frequency.[16] We follow Lin (2010) to define the independent variables used in the model. From Table 1, we can observe that female students have slightly higher GPAs and smoking frequencies than males students. Among the three racial groups

[14]We choose these schools solely based on their sizes – the sizes of their grade 9-12 students range from 100 to 400.

[15]We remove missing observations on dependent variables from the sample and hence there is a subtle difference between the sample size for GPA and smoking.

[16]For example, if a student chooses an answer, "once or twice a week", then we code the variable by $1.5 \times 4 = 6$.

of white, black and other, white students show the highest GPA, as well as the highest smoking frequency, while those of blacks students are the lowest among the three groups. In particular, an average white student smokes 5.286 times per month, while an average black student only smokes 1.84 times per month. For own characteristics, the samples of GPA and smoking show similar patterns thus we focus our discussions on the sample of GPA. About half of the sample are male, and the average age of the whole sample is 15.5. In terms of racial composition, 56% of the sample are white, 31.3% are black, 2.1% are Asian, 5.7% are Hispanic, and 4.9% are of other races. More than 70% of the students live with both parents. About 34.1% of the students have mothers with high school education, 40.7% of the sample have mothers with more than high school education, and 10.1% have mothers with education level less than high school. For mother's occupations, 26.8% of the mothers work at professional jobs, 21.7% of the mothers stay at home, while 34.6% of them work at other jobs. And 1.1% of the mothers receive welfare assistance.

We summarize the average number of nominated friends within and across gender and racial subgroups in Tables 2 and 3. Table 2 shows that on average female and male students nominate similar number of friends. Students have higher tendency to nominate friendships within the same gender. Table 3 shows that on average white students nominate more friends than non-white students. For both whites and blacks, cross-race friendship nominations are much less frequent than intra-race nominations. In contrast, students from other racial groups are mostly likely to nominates whites as friends. A general pattern exhibited in Tables 2 and 3 is that the majority of friendship nominations are formed among people who shares similar characteristics, which points to the importance of controlling for homophily on observed characteristics such as gender and race in the network formation model.

5 Empirical Results

The empirical results for gender and racial peer effects for both GPA and smoking are presented in Tables 4 to 11. For each case, we compare several models without endogenous network formation, Models (1)-(4), with those explicitly model the endogenous network formation process, Models (5)-(8). In particular, for the models without endogenous network formation, we consider the model consists of endogenous social effects only, the model with contextual social effects only, as well as the model includes both social effects, with and without group fixed effects. To capture the endogenous network formation, all network models we consider control for homophily on observed characteristics such as grade, sex and race, as well as unobservable in various dimensions. Specifically, we start from the joint modeling of the SCSAR and the network models with unobservable in one dimension, and then continuously increase the dimension to control for more unobservables, until the estimated peer effect parameters become relatively stable, and the corresponding highest AICM is reached. Then, we can be assured that most significant unobservables affecting both endogenous network formation and outcomes have been effectively controlled for.

5.1 Gender Peer Effects

5.1.1 GPA

From Table 4, we can see that the model with either endogenous effect only, Model (1), or contextual effect only, Model (2), gives very different results than the models with both effects, Models (3) and (4). Both endogenous and contextual effects are shown to be significant in students' academic performance. In addition, several estimated endogenous effects become smaller as the group fixed effects are introduced, indicating the effectiveness of the fixed effect strategy in reducing the contamination bias caused by the unobserved factors facing the group members.

Table 5 shows that several endogenous effect parameters in the SCSAR models drop significantly as more unobservables are controlled for, implying that network formation is endogenous and peer effects estimation would be biased if this feature is not accounted for. In particular, the peer effect parameters slightly drop as one-dimensional unobservable is introduced in Model (5). Although the estimated parameters do not change a lot when the dimension of unobservable is increased to two in Model (6), the substantial increase in the AICM value, compared to its standard error, indicates the great improvement of Model (6). As additional dimension of unobservable is controlled for in Model (7), we can see that some estimated parameters substantially dropped. In particular, the estimated contemporaneous achievement spillovers from same gender friends decreases from 0.136 to 0.109 for females, a significant drop of 25%, and decreases from 0.101 to 0.077 for males, a substantial drop of 31%. Finally, an increase in the dimension of unobservable to four in Model (8) only slightly affects the estimated endogenous effects, yet it causes a drop in the AICM value. Therefore, we take Model (7), which not only includes both endogenous and contextual social effects, and group fixed effects, but also controls for endogenous network formations in terms of observed characteristics of grade, sex and race, as well as unobservable in three dimensions, as our desired model.

For endogenous social effects, an immediate message from Model (7) is that same gender friends generate stronger effects than cross-gender friends, especially for female. In particular, the estimated achievement spillover effect from female friends on females, λ_{ff}, is 0.109 and significant at the 1% level, whereas the effect of male friends is not significant.[17] For male students, the achievement spillover effect from male friends is 0.077, and the estimated effect from female friends, λ_{mf}, is 0.067, both significant at the 1% level, with same gender spillover effect greater than cross-gender effect by 15%. Compared to previous studies which also find stronger effects from same gender friends, such as Kooreman (2007), Nakajima (2007), and Soetevent and Kooreman (2007), and those studies which find that females are more sensitive to peer influences, e.g., Trogdon *et al.* (2008), our results provide more details regarding the mechanism of gender peer effect. Specifically, the peer influences on females are solely generated by their female friends. And compared to males, females receive larger spillover effect from their same gender friends, and they are more influential across gender as they generate larger and significant effect on their male friends. For contextual effects, we find that having older friends, having friends whose mothers achieve less than high school education, or having friends with mothers receive welfare, generate negative effects on a student's GPA, with estimated coefficients of -0.024, -0.173 (5% level), and -0.504 (5% level), respectively. Contrary to several existing studies which find that the proportion of females or blacks generates significant effects on an individual's outcomes, such as Fruehwirth (2013), Hanushek *et al.* (2009), Hoxby (2000), Hansen *et al.* (2006), and Lavy and Schlosser (2011), we find that neither gender nor racial composition of friends generates significant contextual effects on GPA.[18] For own characteristics, most variables have the expected signs. Specifically, male students tend to perform worse than females, and black students tend to score

[17]The indication of the significance level is based on frequentist's approach. It provides a familiar and intuitive interpretation of our results from Bayesian estimation.

[18]Note that for most of the studies aforementioned, endogenous effect is not included in the model. And for Fruehwirth (2013) which includes both endogenous and contextual effects, the endogenous network formation is not fully modeled.

less than whites (10% level). Students who live with both parents, whose mothers have more than high school education, whose mothers work at professional jobs (10% level) tend to perform better.

The parameters from the network formation model show that homophily in terms of observed and unobserved factors are both important for friendship formation. For observed factors, the effects of grade, gender, and race are all significant, with grade effect being the strongest, followed by race and then gender effects. For unobserved factors, all coefficients of distances between unobserved characteristics are negative and significant, implying that the larger the difference between students in terms of unobserved characteristics, the less likely they become friends. Note that not all unobserved factors in the network model contribute to the correction of selection bias in the SCSAR model. For example, the unobserved taste of music may affect students' friendship decisions, while it may not directly affect students' academic outcomes.

5.1.2 Smoking

For smoking, Table 6 shows that Models (1) through (4) generate different estimates, although the distinctions are not as striking as the case of GPA. The group fixed effects slightly reduce the estimated endogenous effects from Model (3) to (4). For the models with endogenous network formation, Table 7 shows that controlling for unobservables does not appear to affect the estimated results of SCSAR models, as evidenced by the small changes in the estimated parameters in Models (5)-(8). We choose Model (7) as our desired model based on the AICM value.

The results in Model (7) show that all four endogenous spillover effects are significant at the 1% level and consistent with the results for GPA, peer influences are stronger intra-gender than inter-gender, and females are more subject to peer effects than males. In particular, for female students, the endogenous effect from female friends is 0.477, which is more than doubled that from male friends, 0.198. For male students, the endogenous effect of same gender friends is 0.318, whereas that of female friends is 0.231, with the former greater than the latter by 38%. Again, females appear to be more influential across gender, with the cross-gender peer effect generated by females larger than that of males by 17%. For contextual effects, friends' age and race turned out to be important. Specifically, having older friends or Asian friends (5% level) significantly decreases smoking frequency of an individual, while having friends of other race increases smoking frequency (10% level). For own characteristics, older students and students of other race (5%

level) tend to smoke more, while black students tend to smoke less. Students living with both parents smoke less, whereas students whose mothers have less than high school education (5% level) or whose mothers receive welfare (5% level) smoke more. Results from the network formation model indicate that, similar to the GPA case, homophily in terms of both observed factors such as grade, race and gender, and unobserved characteristics are significant determinants of friendship formation.

5.2 Racial Peer Effects

5.2.1 GPA

For racial peer effects, nine endogenous effect parameters are specified to capture the within and across racial contemporaneous spillover effects among three groups: white, black and other racial groups. Table 8 shows that for models without endogenous network formation, different model specification gives rise to different estimation results and it is important to include both endogenous and contextual effects, as well as group fixed effect in the model. Similar to the case of gender peer effect for GPA, we can see from Table 9 that controlling for unobservables substantially affect the estimated parameters. The estimated peer effects steadily drop as we increase the dimension of the unobservable from one to two, and the fit of the model improves greatly as demonstrated by the increase in the AICM value. As we continue to increase the dimension to three in Model (7), we can see a significant drop in the estimated parameters, e.g., λ_{ww}, which captures the endogenous spillover effect of white friends on white students, changes from 0.243 to 0.194, a 25% decrease. The AICM value indicates that Model (7) is the desired model.

The results in Model (7) show that white students are most sensitive to peer influences and at the same time, most influential among the three racial groups. In particular, for white students, the endogenous effect generated by their white friends, λ_{ww}, is 0.194, and by their friends from other racial groups, λ_{wo}, is -0.033 (5% level). Besides generating a large intra-race spillover effect, white students also produce a significant endogenous spillover effect on their black friends, with a coefficient of 0.098, as well as an endogenous effect of size 0.139 (5% level) on their friends from other racial groups. For black students, they also receive a marginally significant spillover effect from their same race friends, with an estimated parameter of 0.079 (10% level), which is slightly less than the spillover effect they receive from their white friends, 0.098. On the other hand, black students appear to be the least influential group across race, as they do not generate any significant endogenous effect on their non-black friends. And students from other racial groups appear to interact only with their white friends; they receive an estimated endogenous effect of 0.139 (5% level) from their white friends and produce a negative spillover effect on them. Previous studies, such as Fruehwirth (2013) and Nakajima (2007), can only provide general evidence for strong and significant within-race spillover effects, and insignificant cross-race peer spillover effects.[19] In contrast, our results shed light on the underlying mechanism of racial peer effects. It is worth noting that within-race spillover effects are stronger than inter-race effects for whites, but not for either blacks or students of other racial groups, who receive larger inter-race social interaction effects from their white friends than intra-race interaction effects. In terms of contextual effects, many variables are significant. Specifically, having older friends has a negative effect on GPA, while having friends who are black (5% level), Asian, Hispanic, or of other race has a positive effect. Regarding family background, having friends whose mothers have an education level less than high school (5% level), or whose mothers receive welfare has a negative effect on GPA. For own characteristics, male and black (5% level) students perform worse, while Asian students perform better (10% level). Also students who live both parents, whose mothers with more than high school education, or whose mothers work at professional jobs (10% level) tend to do better in school. Again, the estimates from the network formation model confirms the relevance of homophily on grade, race, gender, as well as unobservables in friendship formation.

5.2.2 Smoking

From Table 10, we can see that Models (1)-(4) give different estimation and the necessity to control for both endogenous and contextual effects, as well as group fixed effect. As in the case of gender peer effect in smoking, Table 11 shows that the estimated peer effect parameters are relatively stable as we increase the dimension of unobservable for endogenous network formation in Models (5)-(8). The AICM value indicates that Model (7) is the desired model.

[19]Note that in Nakajima (2007), only endogenous but not contextual peer effects are included in the model. And in Fruehwirth (2013), the racial groups are divided into two broad groups of white and non-white only. In addition, as mentioned above, the endogenous group formation is not modeled.

Similar to the case of GPA, whites appear to be the racial group who are most subject to the influences of peers and most influential. Specifically, the estimated λ_{ww} and λ_{wo} are 0.511 and 0.127, respectively, with intra-race spillover effect more than four times the magnitude of inter-race effect. In addition, white students generate large effects of 0.382 on their black friends, and of 0.370 on their friends of other racial groups. Interestingly, for students of other racial groups, the inter-race spillover effect generated by their white friends, λ_{ow}, is slightly larger than the intra-race spillover effect, which is 0.321. Again, blacks appear to be the least influential group as they do not generate any significant spillover effect on their friends, even within race. The only social interaction pattern exhibited by black students is that they receive a significant spillover effect from their white friends. For contextual effect, the results show that having younger or Asian friends helps significantly reduce smoking frequency. For own characteristics, older students or students of other race (5% level) tend to smoke more, black students tend to smoke less. Students who live with both parents tend to smoke less, whereas students whose mothers have less than high school education (10% level), or whose mothers receive welfare (5% level) tend to smoke more. And similar pattern for friendship formation is exhibited in the network formation model here.

6 Conclusions

Study of gender and racial peer effects provides insights on the underlying mechanisms of social interactions and sheds light on several policy relevant debates, including those regarding the benefits of single-sex versus coeducational schooling, as well as those on school segregation versus desegregation. However, due to the "reflection problem", most existing studies on gender or racial peer effects have focused on contextual effects, i.e., the effects of gender or racial composition, in particular, female or black proportion, in a group on various outcomes. And endogenous spillover effects have only been examined in several recent papers. The only study that explores both contextual effects of racial composition and contemporaneous achievement spillovers from peers of different races is Fruehwirth (2013). However, possible endogenous formation of peer groups is not fully modeled in that paper.

In this study, we extend the singe order SAR model to higher orders to simultaneously model heterogeneous peer effects from multiple gender and racial groups, and endogenous group formation. We identify not only the effects of gender and racial composition on students' academic achievement and smoking behaviors, but also the contemporaneous spillover effects from the outcomes of peers from difference gender and racial groups. We specify four spatial weighting matrices to capture the contemporaneous spillover effects within and across gender, and nine weighting matrices to capture racial behavior spillover effects.

The results indicate that both endogenous and contextual social effects exist in GPA and smoking, and failing to control for endogenous group formation could seriously bias the estimation results, especially for GPA. Specifically, for endogenous spillover effects along the line of gender, we find that within-gender interaction effects are stronger than cross-gender effects, and compared to males, females are subject to larger within-gender spillover effect and are more influential across gender as they generate larger effect on their male friends. In terms of racial spillover effects, our findings indicate that whites are the racial group who are most sensitive to contemporaneous peer influences and most influential compared to black students and students from other racial groups. Furthermore, for white students, within-race spillover effects are stronger than inter-race effects. While for blacks and students of other racial groups, the inter-race social interaction effects generated by their white friends appear to be slightly stronger than intra-race interaction effects. In addition, black students appear to be the least influential group across race, as they do not generate any significant endogenous effect on their non-black friends. An important policy implication is that policy intervention targeting at female and/or white students will produce exceptionally large effect due to two complimentary mechanisms. One is direct effect, female and/or white students will generate substantial spillover effects on their friends due to their influential status. The other is indirect effect, the direct effect produced by the intervention will be amplified through the feedback effect generated by their friends, as female and/or white students are most responsive to the influences of peers. For contextual effects, we find that peers' age, race and family background such as mother's education and mother's occupation, could generate significant effect on students' GPA and smoking behaviors. Contrary to the findings of several previous studies, we do not find evidence that gender composition of peers significantly affect outcomes such as GPA and smoking. The parameters from the network formation model show that homophily in terms of both observed factors such as grade, race and gender, as well as unobserved characteristics are important for friendship formation.

Appendix A – Identification of the system of network formation and SCSAR models

Our identification strategy is to first provide a heuristic argument on showing that the network formation model is semiparametrically identified, i.e., parameters in the deterministic components as well as distributions of disturbances (including unobservables Z_g and the pure disturbance) are identified. Then, given the identified distributions of disturbances (i.e., can be estimated from the data), we discuss the identification constraints required for the parameters specified for unobsevables Z_g.

Our network model, which is similar to a standard dichotomous choice model, can be motivated from the behavior of utility maximization. For each individual i, he/she chooses $w_{ij,g} = 1$ if $v_{ij,g}(w_{ij,g} = 1) - v_{ij,g}(w_{ij,g} = 0) > 0$ and $w_{ij,g} = 0$ otherwise, where $v_{ij,g}$ stands for i's utility function from the link ij. We can express the above utility difference as

$$v_{ij,g}(w_{ij,g} = 1) - v_{ij,g}(w_{ij,g} = 0) = \mu_{ij,g}(C_g, \gamma) + \xi_{ij,g}(Z_g, \delta), \tag{8}$$

where the deterministic component $\mu_{ij,g}(C_g, \gamma)$ contains $c_{i,g}\gamma_1 + c_{j,g}\gamma_2 + c_{ij,g}\gamma_3$ for the observed exogenous effect. The error component $\xi_{ij,g}(Z_g, \delta)$ contains $\delta_1|z_{i1,g} - z_{j1,g}| + \cdots + \delta_{\bar{d}}|z_{i\bar{d},g} - z_{j\bar{d},g}|$ for homophily on unobserved variables and $\zeta_{ij,g}$ for a pure i.i.d. disturbance.[20] The dichotomous choice model for the network implies the following single index equation,

$$\mathrm{E}(w_{ij,g}|C_g) = P(w_{ij,g} = 1|C_g) = 1 - F_{\xi_{ij,g}}(-\mu_{ij,g}), \tag{9}$$

where $F_{\xi_{ij,g}}(\cdot)$ is the unknown (nonparametric) distribution function of $\xi_{ij,g}$. The identification results in Ichimura (1993) show that parameters in the linear index $\mu_{ij,g}$ are only identified up to a scale and therefore a normalization on one parameter is needed. Also, to achieve identification it requires the existence of at least one continuous exogenous regressor in $\mu_{ij,g}$ whose coefficient is not zero.[21] As the index function is identified, the distribution function $F_{\xi_{ij,g}}$ can be estimated

[20] When $\zeta_{ij,g}$ is treated as the difference of two extreme Type-I distributed disturbances, it results in the logit model in Eq. (5).

[21] The continuous regressors requirement is for convenience purpose but not necessary. But with the presence of all discrete regressors, some patterns of combinations of discrete values are needed for identification. With a parametric function for $\xi_{ij,g}$, all of those are not needed in general.

from the data by a nonparametric kernel regression with the index $\mu_{ij,g}$ as the regressor. Since the disturbances are continuously distributed, by assuming that $\mu_{ij,g}$ can take on values which cover the support of the probability density $f_{\xi_{ij,g}}$, $f_{\xi_{ij,g}}$ and moments of $\xi_{ij,g}$ are also estimable from the data.

From central moments of $\xi_{ij,g}$, we can study the identification constraints required for the coefficients of unobsevables Z_g. We temporarily suppress the group subscript for simplicity. First we consider a case where unobservables Z are in one dimension, i.e., $\xi_{ij}(Z,\delta) = \delta_1|z_{i1} - z_{j1}| + \zeta_{ij}$. The variance (second order central moment) of ξ_{ij} is equal to $\delta_1^2 \text{Var}(|z_{i1} - z_{j1}|) + \sigma_\zeta^2$, where σ_ζ^2 is the variance of ζ_{ij} and is normalized for the arbitrary scaling problem in discrete choice models. Because Z is unobserved, in order to obtain δ_1 alone, we normalize the variance of Z to one so that $\text{Var}(|z_{11} - z_{21}|)$ is a known value. This normalization is required for every dimension of Z we consider.

Next we consider the case where unobservables Z are in two dimensions, i.e., $\xi_{ij}(Z,\delta) = \delta_1|z_{i1} - z_{j1}| + \delta_2|z_{i2} - z_{j2}| + \zeta_{ij}$. Note that under multiple dimensions, we need the independence assumption between Z in different dimensions. Otherwise, unknown correlations between Z in different dimensions will make identifying δ_d's from central moments of ξ_{ij} impossible. The variance of ξ_{ij}, with Z in two dimensions, is $(\delta_1^2 + \delta_2^2)\text{Var}(|z_{i1} - z_{j1}|) + \sigma_\zeta^2$. From it, We cannot separately identify δ_1 and δ_2. Thus, we need to look for other identification conditions from higher order central moments of ξ_{ij}. For example, in the third order central moment, we can obtain $(\delta_1^3 + \delta_2^3)t + \text{E}[(\zeta_{ij} - \text{E}(\zeta_{ij}))^3]$, where $t = \text{E}[(|z_{i1} - z_{j1}| - \text{E}(|z_{i1} - z_{j1}|))^3]$. In order to identify δ_1 and δ_2, we need to separate t from $(\delta_1^3 + \delta_2^3)$, which requires t to be a known value. In other words, unobservables Z need to be from a known distribution, e.g., normal distribution. Similarly, we need to assume ζ_{ij} to be from a known distribution, e.g., logistic distribution for knowing the value of $\text{E}[(\zeta_{ij} - \text{E}(\zeta_{ij}))^3]$. We can obtain other polynomial equations involving δ_1 and δ_2 from fourth and higher order central moments of ξ_{ij}. Eventually, the system of these polynomial equations can be used to solve values of δ_1 and δ_2. The only problem remained is that values of δ_1 and δ_2 can be arbitrarily switched. To avoid this problem, we require $|\delta_1| \geq |\delta_2|$. When Z has $\bar{d}$ dimensions, we will then require $|\delta_1| \geq |\delta_2| \geq \cdots \geq |\delta_{\bar{d}}|$. The implication of this constraint is that z_{i1} ($z_{i\bar{d}}$) represents the dimension of Z which has the greatest (smallest) influence on friendship formation. Similarly, the identification of δ_d's when Z are in three or higher dimensions can be retrieved from the second to higher order central moments of ξ_{ij}.

We next look at the outcome equation represented by the SCSAR model of Eq. (4). We first use the difference approach to eliminate the group fixed effect in the SCSAR model. The variables Y_g, W_gY_g, X_g, W_gX_g, Z_g, and u_g are transformed to $Y_g^* = T_gY_g$, $(W_gY_g)^* = T_g(W_gY_g)$, $X_g^* = T_gX_g$, $(W_gX_g)^* = T_g(W_gX_g)$, $Z_g^* = T_gZ_g$, and $u_g^* = T_gu_g$ with a $(m_g-1)\times m_g$ matrix

$$T_g = \begin{pmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & -1 & 1 & \\ & & & \ddots & \ddots \end{pmatrix}.$$

After the transformation, we obtain

$$Y_g^* = \lambda(W_gY_g)^* + X_g^*\beta_1 + (W_gX_g)^*\beta_2 + Z_g^*\rho + u_g^*, \;\; g = 1,\cdots,G. \tag{10}$$

Next, taking an expectation of Eq. (10) conditional on W_g, we have

$$\mathrm{E}(Y_g^*|W_g) = \lambda\mathrm{E}((W_gY_g)^*|W_g) + \mathrm{E}(X_g^*|W_g)\beta_1 + \mathrm{E}((W_gX_g)^*|W_g)\beta_2 + \mathrm{E}(Z_g^*|W_g)\rho, \tag{11}$$

for $g = 1,\cdots,G$. Note that, $\mathrm{E}(Y_g^*|W_g)$, $\mathrm{E}((W_gY_g)^*|W_g)$, $\mathrm{E}(X_g^*|W_g)$, $\mathrm{E}((W_gX_g)^*|W_g)$, and $\mathrm{E}(Z_g^*|W_g)$ in Eq. (11) can be identified from the data. Especially, we can identify $\mathrm{E}(Z_g^*|W_g) = \int_z Z_g^* P(Z_g|W_g)dZ_g = \int_z Z_g^* \frac{P(Z_g)P(W_g|Z_g)}{P(W_g)}dZ_g$ given that parameters in $P(W_g|Z_g)$ are identified (estimated) from the network model. With a slight abuse on notations, we let $\mathrm{E}((WY)^*|W)$, $\mathrm{E}(X^*|W)$, $\mathrm{E}((WX)^*|W)$ and $\mathrm{E}(Z^*|W)$ without the subscript g denote observations stacked across groups.

Let $\mathbb{T} = [\mathrm{E}((WY)^*|W),\ \mathrm{E}(X^*|W),\ \mathrm{E}((WX)^*|W),\ \mathrm{E}(Z^*|W)]$, the condition that $\mathbb{T}'\mathbb{T}$ has a full column rank will identify the parameters in Eq. (11). One more thing to note is that as $P(W_g|Z_g)$ is the same for Z_g and $-Z_g$, plus $P(Z_g)$ is a symmetric distribution, we have $\mathrm{E}(Z_g^*|W_g) = \mathrm{E}(-Z_g^*|W_g)$. Therefore, for identification we need to normalize ρ to be positive.

Appendix B – Bayesian Estimation of the high order SCSAR model and the network model

To estimate the high order SCSAR model for continuous variables, we follow closely to the MCMC sampling procedure described in Hsieh and Lee (2013). One major difference is that candidate values of endogenous effect parameters are rejected during the Metropolis-Hastings step if they violate the stationary condition required by the high order SCSAR model, i.e., $|\lambda_{11,c}|+\cdots+|\lambda_{\bar{p}\bar{p},c}|<1$, as suggested by LeSage and Pace (2009).

The Tobit-type SCSAR model has not been studied in previous studies, thus we describe the estimation procedure in details. First, we divide m_g individuals in group g into two blocks, such that the first m_{g1} individuals have $y_{i,tg}$ equals zero and the remaining individuals who are arranged from $m_{g1}+1$ to m_g have a positive $y_{i,tg}$. Then, Eq. (3) can be conformably decomposed into

$$\begin{pmatrix} \ddot{Y}_{tg1} \\ Y_{tg2} \end{pmatrix} = \lambda_{11,t} \begin{pmatrix} \widetilde{W}_{11,g}^{11} & \widetilde{W}_{11,g}^{12} \\ \widetilde{W}_{11,g}^{21} & \widetilde{W}_{11,g}^{22} \end{pmatrix} \begin{pmatrix} Y_{tg1} \\ Y_{tg2} \end{pmatrix} + \cdots + \lambda_{\bar{p}\bar{p},t} \begin{pmatrix} \widetilde{W}_{\bar{p}\bar{p},g}^{11} & \widetilde{W}_{\bar{p}\bar{p},g}^{12} \\ \widetilde{W}_{\bar{p}\bar{p},g}^{21} & \widetilde{W}_{\bar{p}\bar{p},g}^{22} \end{pmatrix} \begin{pmatrix} Y_{tg1} \\ Y_{tg2} \end{pmatrix}$$
$$+ \begin{pmatrix} X_{1g} \\ X_{2g} \end{pmatrix} \beta_{1t} + \begin{pmatrix} \widetilde{W}_{g}^{11} & \widetilde{W}_{g}^{12} \\ \widetilde{W}_{g}^{21} & \widetilde{W}_{g}^{22} \end{pmatrix} \begin{pmatrix} X_{1g} \\ X_{2g} \end{pmatrix} \beta_{2t} + \begin{pmatrix} l_{1g} \\ l_{2g} \end{pmatrix} \alpha_{tg} + \begin{pmatrix} \varepsilon_{tg1} \\ \varepsilon_{tg2} \end{pmatrix}, \tag{12}$$

where $Y_{tg2}>0$ and $Y_{tg1}=0$ with the corresponding latent variables, $\ddot{Y}_{tg1}\leq 0$. The joint probability of Y_{tg} and W_g is

$$\begin{aligned} & P(Y_{tg}, W_g | X_g, C_g, \theta_t, \alpha_{tg}) \\ &= \int_{Z_g} P(Y_{tg1}=0, Y_{tg2} | W_g, X_g, Z_g, \theta_t, \alpha_{tg}) \cdot P(W_g | C_g, Z_g, \theta_t) \cdot f(Z_g) \cdot dZ_g \\ &= \int_{Z_g} \left(\int_{\ddot{Y}_{tg1}} I(Y_{tg1}=0, \ddot{Y}_{tg1}) \cdot P(\ddot{Y}_{tg1}, Y_{tg2} | W_g, X_g, Z_g, \theta_t, \alpha_{tg}) \cdot d\ddot{Y}_{tg1} \right) \cdot P(W_g | C_g, Z_g, \theta_t) \cdot f(Z_g) \cdot dZ_g \\ &= \int_{Z_g} \left(\int_{-\infty}^{-A_g} \left| I_{m_g - m_{g1}} - \lambda_{11,t} \widetilde{W}_{11,g}^{22} - \cdots - \lambda_{\bar{p}\bar{p},g} \widetilde{W}_{\bar{p}\bar{p},g}^{22} \right| \cdot f(\varepsilon_{tg1}, \varepsilon_{tg2} | W_g, X_g, Z_g, \theta_t, \alpha_{tg}) \cdot d\varepsilon_{tg1} \right) \cdot \\ &\qquad P(W_g | C_g, Z_g, \theta_t) \cdot f(Z_g) \cdot dZ_g, \end{aligned} \tag{13}$$

where $\theta_t = (\gamma', \delta', \lambda_t', \beta_{1t}', \beta_{2t}', \sigma_u^2, \rho')$; $I(Y_{tg1}=0, \ddot{Y}_{tg1})$ is a dichotomous indicator which equals 1 when $\ddot{Y}_{tg1}$ is negative, and 0, otherwise; and

$A_g = \left(\lambda_{11,t} \widetilde{W}_{11,g}^{22} Y_{tg2} + \cdots + \lambda_{\bar{p}\bar{p},t} \widetilde{W}_{\bar{p}\bar{p},g}^{22} Y_{tg2} + X_{1g}\beta_{1t} + (\widetilde{W}_g^{11} X_{1g} + \widetilde{W}_g^{12} X_{2g})\beta_{2t} + l_{1g}\alpha_{tg} \right)$.

We specify the prior distributions of θ_t, unobserved characteristics $\{z_{i,g}\}$, and group effects

$\{\alpha_{tg}\}$ as follows:

$$z_{i,g} \sim \mathcal{N}_{\bar{d}}(0, I_{\bar{d}}), \ \ i = 1, \cdots, m_g; \ g = 1, \cdots, G, \tag{14}$$

$$\phi = (\gamma', \delta') \sim \mathcal{TN}_{\bar{q}+\bar{d}}(\phi_0, \Phi_0), \tag{15}$$

$$\lambda_{pq,t} \sim U[-\tau, \tau], \ \ p, q = 1, \cdots, \bar{p}, \tag{16}$$

$$\beta_t = (\beta'_{1t}, \beta'_{2t}) \sim \mathcal{N}_{2k}(\beta_0, B_0), \tag{17}$$

$$\sigma_u^2 \sim \mathcal{IG}(\frac{\kappa_0}{2}, \frac{\nu_0}{2}), \tag{18}$$

$$\rho \sim \mathcal{TN}_{\bar{d}}(\rho_0, \rho_0), \tag{19}$$

$$\alpha_{tg} \sim \mathcal{N}(\alpha_0, A_0), \ \ g = 1, \cdots, G, \tag{20}$$

where $\mathcal{TN}$ represents a truncated normal distribution and $\mathcal{IG}$ represents an inverse Gamma distribution. The coefficients γ and δ in the function $\psi_{ij,g}$ of Eq. (5) are grouped into ϕ with the truncated normal prior to reflect the identification constraint $|\delta_1| \geq |\delta_2| \geq \cdots \geq |\delta_{\bar{d}}|$. For endogenous effects λ_t, we employ uniform priors between $-\tau$ to τ.[22] For coefficients ρ, we also employ a truncated normal prior to reflect the identification constraint $\rho \geq 0$. The rest are the commonly used conjugate priors in the Bayesian literature.

We include the sampling of latent variables, $\ddot{Y}_{tg1}$, during the MCMC procedure along with other unobservables as an augmentation (Albert and Chib, 1993). By Bayes' theorem, the joint posterior distribution of the parameters and unobservables in the model is[23]

$$\begin{aligned} & P\left(\theta_t, \{\alpha_{tg}\}, \{\ddot{Y}_{tg1}\}, \{Z_g\} | \{Y_{tg}\}, \{W_g\}\right) \\ & \propto \pi(\theta_t, \{Z_g\}, \{\alpha_{tg}\}) \cdot \prod_{g=1}^{G} \left\{ \left(\prod_{i=1}^{m_{g1}} I(y_{i,tg} = 0) \cdot I(\ddot{y}_{i,tg} \leq 0) \right) \cdot P\left(Y_{tg}, W_g, \ddot{Y}_{tg1} | Z_g, \theta_t, \alpha_{tg}\right) \right\}, \end{aligned} \tag{21}$$

where $\pi(\cdot)$ represents the density function of the prior distribution with independent variables $\{X_g\}$ and $\{C_g\}$ suppressed from the above expression for simplicity. We assume independence between prior distributions of all unobservables. Applying the Gibbs sampling, we simulate random draws

[22] The value of τ should reflect the stationary condition required by the high order SCSAR model for Tobit-type variables, which is $|I_{m_g - m_{g1}} - \lambda_{11,t}\widetilde{W}^{22}_{11,g} - \cdots - \lambda_{\bar{p}\bar{p},g}\widetilde{W}^{22}_{\bar{p}\bar{p},g}| > 0$. Due to the restriction imposed on the support of prior distributions, we reject Metropolis-Hastings candidate values of $\{\lambda_{pq,t}\}$ which violate this stationary condition during the posterior simulation.

[23] We simplify the notation $\{A_g\}_{g=1}^{G}$ to $\{A_g\}$ to represent the collection of A_g across G groups.

from this joint posterior distribution by iteratively drawing samples from conditional posterior distributions for each parameter groups. Here we list the set of conditional posterior distributions required by the Gibbs sampler:

(i) $P\left(\ddot{Y}_{tg1} \middle| Y_{tg}, W_g, \theta_t, \alpha_{tg}, Z_g\right)$, $g = 1, \cdots, G$.

By applying Bayes' theorem, we have

$$P\left(\ddot{Y}_{tg1} \middle| Y_{tg}, W_g, \theta_t, \alpha_{tg}, Z_g\right) \propto \left(\prod_{i=1}^{m_{g1}} I(y_{i,tg} = 0) I(\ddot{y}_{it,g} \leq 0)\right) P(\ddot{Y}_{tg1}, Y_{tg}, W_g | \theta_t, \alpha_{tg}, Z_g). \quad (22)$$

(ii) $P(z_{i,g} | \ddot{Y}_{tg1}, Y_{tg}, W_g, \theta_t, \alpha_{tg}, Z_{-i,g})$, $i = 1, \cdots, m_g$, $g = 1, \cdots, G$.

By applying Bayes' theorem,

$$P(z_{i,g} | \ddot{Y}_{tg1}, Y_{tg}, W_g, \theta_t, \alpha_{tg}, Z_{-i,g}) \propto \mathscr{N}_{\bar{d}}(z_{i,g}; 0, I_{\bar{d}}) \cdot P(\ddot{Y}_{tg1}, Y_{tg}, W_g | \theta_t, \alpha_g, Z_g), \quad (23)$$

where $\mathscr{N}_{\bar{d}}(.; 0, I_{\bar{d}})$ is the multivariate normal prior distribution of $z_{i,g}$.

(iii) $P(\phi | \{W_g\}, \{Z_g\})$.

By applying Bayes' theorem, we have

$$P(\phi | \{W_g\}, \{Z_g\}) \propto \mathscr{T}\mathscr{N}_{\bar{q}+\bar{d}}(\phi; \phi_0, \Phi_0) \cdot \prod_{g=1}^{G} P(W_g | Z_g, \phi), \quad (24)$$

where $\mathscr{T}\mathscr{N}_{\bar{q}+\bar{d}}(\phi; \phi_0, \Phi_0)$ is the truncated normal prior distribution of ϕ.

(iv) $P(\lambda_{pq,t} | \{\ddot{Y}_{tg1}\}, \{Y_{tg}\}, \{W_g\}, \beta_t, \sigma_u^2, \rho, \{\alpha_{tg}\}, \{Z_g\})$, $p, q = 1, \cdots, \bar{p}$.

By applying Bayes' theorem, we have

$$P(\lambda_{pq,t} | \{\ddot{Y}_{tg1}\}, \{Y_{tg}\}, \{W_g\}, \beta_t, \sigma_u^2, \rho, \{\alpha_{tg}\}, \{Z_g\}) \propto \prod_{g=1}^{G} P(\ddot{Y}_{tg1}, Y_{tg} | W_g, \lambda_t, \beta_t, \sigma_u^2, \rho, \alpha_{tg}, Z_g), \quad (25)$$

where $\lambda_{pq,t} \in [-\tau, \tau]$.

(v) $P(\beta_t | \{\ddot{Y}_{tg1}\}, \{Y_{tg}\}, \{W_g\}, \lambda_t, \sigma_u^2, \rho, \{\alpha_{tg}\}, \{Z_g\})$.

By applying Bayes' theorem, we have

$$\begin{aligned} &P(\beta_t | \{\ddot{Y}_{tg1}\}, \{Y_{tg}\}, \{W_g\}, \lambda_t, \sigma_u^2, \rho, \{\alpha_{tg}\}, \{Z_g\}) \\ &\quad \propto \mathscr{N}_{2k}(\beta_t; \beta_0, B_0) \cdot \prod_{g=1}^{G} P(\ddot{Y}_{tg1}, Y_{tg} | W_g, \lambda_t, \beta_t, \sigma_u^2, \rho, \alpha_{tg}, Z_g). \end{aligned}$$

Since both $\mathcal{N}_{2k}(\beta_t;\beta_0,B_0)$ and $P(\ddot{Y}_{tg1},Y_{tg}|W_g,\lambda_t,\beta_t,\sigma_u^2,\rho,\alpha_{tg},Z_g)$ are in terms of normal density, we obtain the standard linear model results in which

$$\begin{aligned}
&P(\beta_t|\{\ddot{Y}_{tg1}\},\{Y_{tg}\},\{W_g\},\lambda_t,\sigma_u^2,\rho,\{\alpha_{tg}\},\{Z_g\}) \propto \mathcal{N}_{2k}\left(\beta_t;\hat{\beta}_t,\mathbf{B}_t\right)\\
&\hat{\beta}_t = \mathbf{B}_t\left(B_0^{-1}\beta_0 + \sum_{g=1}^{G}\mathbf{X}_g'(\sigma_u^2 I_{m_g})^{-1}(S_{tg}[\ddot{Y}_{tg1}',Y_{tg2}']' - Z_g\rho - l_g\alpha_{tg})\right)\\
&\mathbf{B}_t = \left(B_0^{-1} + \sum_{g=1}^{G}\mathbf{X}_g'(\sigma_u^2 I_{m_g})^{-1}\mathbf{X}_g\right)^{-1},
\end{aligned} \tag{26}$$

where $\mathbf{X}_g = (X_g,\widetilde{W}_g X_g)$ and $S_{tg} = (I_{m_g} - \lambda_{11,t}\widetilde{W}_{11,g} - \cdots - \lambda_{\bar{p}\bar{p},t}\widetilde{W}_{\bar{p}\bar{p},g})$.

(vi) $P(\sigma_u^2|\{\ddot{Y}_{tg1}\},\{Y_{tg}\},\{W_g\},\lambda_t,\beta_t,\rho,\{\alpha_{tg}\},\{Z_g\})$.

By applying Bayes' theorem, we have

$$\begin{aligned}
&P(\sigma_u^2|\{\ddot{Y}_{tg1}\},\{Y_{tg}\},\{W_g\},\lambda_t,\beta_t,\rho,\{\alpha_{tg}\},\{Z_g\})\\
&\quad\propto \mathcal{IG}\left(\sigma_u^2;\frac{\kappa_0}{2},\frac{\nu_0}{2}\right)\prod_{g=1}^{G}P(\ddot{Y}_{tg1},Y_{tg}|W_g,\lambda_t,\beta_t,\sigma_u^2,\rho,\alpha_{tg},Z_g)\\
&\quad\propto \mathcal{IG}\left(\sigma_u^2;\frac{\kappa_0+\sum_{g=1}^{G}m_g}{2},\frac{\nu_0+\sum_{g=1}^{G}u_{tg}'u_{tg}}{2}\right),
\end{aligned} \tag{27}$$

where $u_{tg} = S_{tg}[\ddot{Y}_{tg1}',Y_{tg2}']' - X_g\beta_{1t} - \widetilde{W}_g X_g\beta_{2t} - Z_g\rho - l_g\alpha_{tg}$.

(vii) $P(\rho|\{\ddot{Y}_{tg1}\},\{Y_{tg}\},\{W_g\},\lambda_t,\beta_t,\sigma_u^2,\{\alpha_{tg}\},\{Z_g\})$.

By applying Bayes' theorem, we have

$$\begin{aligned}
&P(\rho|\{\ddot{Y}_{tg1}\},\{Y_{tg}\},\{W_g\},\lambda_t,\beta_t,\sigma_u^2,\{\alpha_{tg}\},\{Z_g\})\\
&\quad\propto \mathcal{TN}_{\bar{d}}(\rho;\rho_0,\rho_0)\prod_{g=1}^{G}P(\ddot{Y}_{tg1},Y_{tg}|W_g,\lambda_t,\beta_t,\sigma_u^2,\rho,\alpha_{tg},Z_g),
\end{aligned} \tag{28}$$

where $\mathcal{TN}_{\bar{d}}(\rho;\rho_0,\rho_0)$ is the truncated normal prior distribution of ρ.

(viii) $P(\alpha_{tg}|\ddot{Y}_{tg1},Y_{tg},W_g,\lambda_t,\beta_t,\sigma_u^2,\rho,Z_g)$, $g = 1,\cdots,G$

By applying Bayes' theorem, we have

$$P(\alpha_g|\ddot{Y}_{tg1},Y_{tg},W_g,\lambda_t,\beta_t,\sigma_u^2,\rho,Z_g) \propto \mathcal{N}(\alpha_g;\alpha_0,A_0)\cdot P(\ddot{Y}_{tg1},Y_{tg}|W_g,\lambda_t,\beta_t,\sigma_u^2,\rho,\alpha_{tg},Z_g). \tag{29}$$

Similar to (v), we can further obtain

$$
\begin{aligned}
&P(\alpha_g|\ddot{Y}_{tg1}, Y_{tg}, W_g, \lambda_t, \beta_t, \sigma_u^2, \rho, Z_g) \propto \mathcal{N}(\alpha_g; \hat{\alpha}_g, R_g), \\
&\hat{\alpha}_g = R_g(A_0^{-1}\alpha_0 + l_g'(\sigma_u^2 I_{m_g})^{-1}(S_{tg}[\ddot{Y}_{tg1}', Y_{tg2}']' - \mathbf{X}_g\beta_t - Z_g\rho)), \\
&R_g = (A_0^{-1} + l_g(\sigma_u^2 I_{m_g})^{-1} l_g')^{-1}. \qquad (30)
\end{aligned}
$$

Appendix C – Derivation of the AICM

The conventional AIC (Akaike 1973) is defined as

$$\text{AIC} = 2\ell_{\max} - 2d, \qquad (31)$$

where $\ell_{\max}$ is the maximum loglikelihood and d is the dimension of the parameters in the model. $\ell_{\max}$ is not directly observable in Bayesian estimation approach because $\ell_{\max}$ may not be reached during the MCMC sampling procedure; however, following Raftery *et al.* (2007), it may be estimated given the posterior distribution of the loglikelihoods,

$$\ell_{\max} - \ell_t \sim \text{Gamma}(d/2, 1), \qquad (32)$$

where $\{\ell_t : t = 1, \cdots, T\}$ is a sequence of loglikelihoods from MCMC posterior draws with a proper thinning so that they are approximately independent. The distributional assumption in Eq. (32) is asymptotically evident when the amount of data underlying the likelihoods increases to infinity (Bickel and Ghosh 1990; Dawid 1991). Based on the Gamma distribution, we know $E[\ell_{\max} - \ell_t] = d/2$ and $\text{Var}(\ell_t) = d/2$. Therefore, we can obtain the moment estimators $\hat{d} = 2s_\ell^2$ and $\hat{\ell}_{\max} = \bar{\ell} + s_\ell^2$, where $\bar{\ell}$ and s_ℓ^2 are the sample mean and variance of the ℓ_t's, respectively. The simulation-based (Monte Carlo) version of AIC is given as

$$\text{AICM} = 2\hat{\ell}_{\max} - 2\hat{d} = 2(\bar{\ell} - s_\ell^2). \qquad (33)$$

and its standard error can be calculated by

$$\text{S.E.}(\text{AICM}) = \sqrt{4\hat{d}/(2T) + 4\hat{d}(11\hat{d}/4 + 12)/T} \qquad (34)$$

by using the fact that $\text{Var}(\bar{\ell}) \approx d/(2T)$ and $\text{Var}(s_\ell^2) \approx d(11d/4 + 12)/T$ and the approximate independence between $\bar{\ell}$ and s_ℓ^2.

References

Akaike, H. (1973). Information theory and an extension of the maximum likelihood principle. In B. N. Petrov and F. Csaki (Eds.), Second international symposium on information theory, 267-281. Budapest: Academiai Kiado.

Albert, J. H. and Chib, S. (1993). Bayesian analysis of binary and polychotomous response data. *Journal of American Statistical Association*, 88, 669-679.

Angrist, J. D. and Lang, K. (2004). Does school integration generate peer effects? Evidence from Boston's metco program. *American Economic Review*, 94, 1613-1634.

Anselin, L. and Bera, A. (1998) Spatial dependence in linear regression models, with an introduction to spatial econometrics. In *Handbook of Applied Economic Statistics* (eds) A. Ullah and D.E.A. Giles. Marcel Dekker.

Bickel, P. J. and Ghosh, J. K. (1990). Decomposition of the likelihood ratio statistic and the Bartlett correction V A Bayesian argument. *Annals of Statistics*, 18, 1070V1090.

Blommestein, H.J. (1983) Specification and estimation of spatial econometric models: A discussion of alternative strategies for spatial economic modelling. *Regional Science and Urban Economics*, 13, 250V271.

Blommestein, H.J. (1985) Elimination of circular routes in spatial dynamic regression equations. *Regional Science and Urban Economics* 15, 121V130.

Bobonis, G. and Finan, F. (2009). Neighborhood peer effects in secondary school enrollment decisions. *The Review of Economics and Statistics*, 91, 695-716.

Boozer, M. A., Krueger, A. B., and Wolkon, S. (1992). Race and school quality since Brown v. Board of Education. Working paper.

Bramoullé, Y., Djebbari, H., and Fortin, B. (2009), "Identification of Peer Effects through Social Networks," *Journal of Econometrics*, 150, 41-55.

Calvó-Armengol, A., Patacchini, E., and Zenou, Y. (2009). "Peer Effects and Social Networks in Education," *Review of Economic Studies*, 76, 1239-1267.

Chib, S. (1995). Marginal likelihood from the Gibbs output. *Journal of American Statistical Association*, 90, 1313-1321.

Chib, S. and Carlin, B. (1999). On MCMC sampling in hierarchical longitudinal models. *Statistics and Computing*, 9, 17-26.

Chib, S. and Jeliazkov, I. (2001). Marginal likelihood from the Metropolis-Hastings output. *Journal of the American Statistical Association*, 96, 270-281.

Christofides, L. N., Hoy, M., Milla, J. and Stengos, T. (2011). The Implication of Peer and Parental Influences on University Attendance: A Gender Comparison. Working paper, University of Guelph, Guelph, ON, Canada.

Clark, A. and Loheac, Y. (2007). It wasn't me. It was them! Social influence in risky behaviors among young adults, *Journal of Health Economics*, 26, 763-784.

Cook, M. D. and Evans, W. N. (2000). Families or schools? Explaining the convergence in white and black academic performance. *Journal of Labor Economics*, 18, 729-754.

Dawid, A. P. (1991). Fisherian inference in likelihood and prequential frames of reference. *Journal of Royal Statistical Society B*, 53, 79V109, (with discussion).

Duflo, E. and Saez, E. (2002). Participation and investment decision in a retirement plan: the influence of colleagues' choices. *Journal of Public Economics* 85, 121-148.

Echenique, F., Fryer, R.G. and Kaufman, A. (2006). Is school segregation good or bad? *American Economic Review* 96, (2), 265-269.

Evans, W. N., W. E. Oates and R. M. Schwab (1992). Measuring peer group effects: A study of teenage behavior. *Journal of Political Economy* 100, 966-91.

Fletcher, J. M. (2010). Social interactions and smoking: Evidence using multiple student coherts, instrumental variables, and school fixed effects. *Health Economics* 19, 466-484.

Fletcher, J. M. (2011). Peer influences on adolescent alcohol consumption: evidence using instrumental variables/fixed efect approach. *Journal of Population Economics*, 25, 1265-1286.

Fortin, B., and Yazbeck, M. (2011). Peer effects, fast food consumption and adolescent weight gain. CIRANO - Scientific Publications 2011s-20. Available at SSRN: http://ssrn.com/abstract=1759978 or http://dx.doi.org/10.2139/ssrn.1759978

Fruehwirth, J. C. (2010). Classroom peer effects. *The New Palgrave Dictionary of Economics*, Eds. Steven N. Durlauf and Lawrence E. Blume, Palgrave Macmillan.

Fruehwirth, J. C. (2013). Identifying Peer Achievement Spillovers: Implications for Desegregation and the Achievement Gap. *Quantitative Economics*, forthcoming.

Gaviria, A. and S. Raphael (2001). School-based peer effects and juvenile behaviors. *Review of Economics and Statistics* 83, 257-268.

Han, L. and Li, T. (2009). The gender difference of peer influence in higher education. *Economics of Education Review*, 28, 129-134.

Handcock, M. S., Raftery, A. E., and Tantrum, J. M. (2007). Model-based clustering for social networks. *Journal of the Royal Sataistical Society* 170, 301-54.

Hansen, Z., Owan, H. and Pan, J. (2006). The Impact of Group Diversity on Performance and Knowledge Spillover – An Experiment in a College Classroom. NBER Working Paper No. 12251.

Hanushek, E. A., Kain, J. F., Rivkin, J. M., and Rivkin, S. G. (2003). Does peer ability affect student achievement? *Journal of Applied Econometrics* 18, 527-544.

Hanushek, E. A. and Raymond, M. E. (2005). Does school accountability lead to improved student performance? *Journal of Policy Analysis and Management*, 24, 297-327.

Hanushek, E. A., Kain, J. F., and Rivkin, S. G. (2009). New evidence about Brown v. Board of Education: The complex effects of school racial composition on achievement. *Journal of Labor Economics*, 27, 349-383.

Haynie, D. L. (2001). Delinquent peers revisited: A network approach for understanding adolescent delinquency. *American Journal of Sociology*, 106, 1013-1057.

Haynie, D. L. (2002). Friendship networks and deliquency: The relative nature of peer deliquency. *Journal of Quantitative Criminology*, 18, 99-134.

Heckman, J. J. (1979). Sample selection bias as a specification error. *Econometrica: Journal of the econometric society*, 153-161.

Hoxby, D. L. (2000). Peer effects in the classroom: Learning from gender and race variation. NBER working paper 7867.

Hsieh, C. S. and Lee, L. F. (2013), "A Social Interaction Model with Endogenous Friendship Formation and Selectivity," Working paper.

Huang, J.S. (1984). The autoregressive moving average model for spatial analysis. *Australian Journal of Statistics* 26, 169V178.

Kandel, D.B., Kiros, G.E. and Hu, M.C. (2004). Racial/ethnic differences in cigarette smoking initiation and progression to daily smoking: A multilevel analysis. *American Journal of Public Health*, 94(1), 128-35.

Kooreman, P. (2007). Time, money, peers, and parents: Some data and theories on teenage behavior. *Journal of Population Economics* 20, 9-33.

Goldsmith-Pinkham P., Imbens G. W. 2013. Social networks and the identification of peer effects. *Journal of Businees and Economic Statistics*, 31, 253-264.

Grogger, J. T. (1996). Does school quality explain the recent black/white wage trend *Journal of Labor Economics* 14 (2), 231-253.

Lavy, V. and Schlosser, A. (2011). Mechanisms and impacts of gender peer effects at school. *American Economic Journal: Applied Economics*, 3, 1-33.

Lalive, R. and Cattaneo, M. A. (2009). Social interactions and schooling decisions. *The Review of Economics and Statistics*, 91, 457-477.

LeSage, J. P. and Pace, R. K. (2009). *Introduction to Spatial Econometrics*, CRC Press.

Lee, L. F., and Liu, X. (2010). Efficient GMM estimation of high order spatial autoregressive models with autoregressive disturbances. *Econometric Theory* 26, 187-230.

Lee, L. F., Liu, X., and Lin, X. (2010), "Specification and Estimation of Social Interaction Models with Network Structures," *The Econometrics Journal*, 13, 145-76.

Lin, Xu (2010). Identifying peer effects in student academic achievement by spatial autoregressive models with group unobservables. *Journal of Labor Economics* 28, 825-860.

Lundborg, P. (2006). Having the wrong friends? Peer effects in adolescent substance use. *Journal of Health Economics* 25, 214-233.

Manski, C. F. (1993). Identification of endogneous social effects: The reflection problem. *Review of Economic Studies* 60, 531-542.

Moffitt, R. A. (2001), "Policy Interventions, Low-Level Qquilibria, and Social Interactions," In *Social Dynamics* Eds. Durlauf, S., and H. P. Young, MIT Press, Combridge, MA, 45-82.

Nakajima, R. (2007). Measuring peer effects on youth smoking behavior. *Review of Economic Studies*, 74, 897-935.

Navarro S. 2008. Control functions. *The New Palgrave Dictionary of Economics* Second Edition. Eds. Steven N. Durlauf and Lawrence E. Blume. Palgrave Macmillan.

Nathan, A. (2008). The effects of racial and extracurricular friendship diversity on achievement. Working Papers 0816, College of the Holy Cross, Department of Economics.

Olsen, R. J. (1980). A least squares correction for selectivity bias. *Econometrica: Journal of the Econometric Society*, 1815-1820.

Powell, L. M., J. A. Tauras, and H. Ross (2005). The importance of peer effects, cigarette prices and tobacco control policies for youth smoking behavior. *Journal of Health Economics* 24, 950-968.

Raftery, A. E., Newton, M. A., Satagopan, J. M., Krivitsky, P. N. (2007). Estimating the integrated likelihood via posterior simulation using the harmonic mean identity. *Bayesian Statistics*, 8, 1-45.

Rivkin, S. G. (2000). School desegregation, academic attainment, and earnings. *Journal of Human Resources*, 35, 333-346.

Rivkin, S. G. (2001). Tiebout sorting, aggregation and the estimation of peer group effects. *Economics of Education Review* 20, 201-09.

Sacerdote, B. (2001). Peer effects with random assignment: Results for Dartmouth roommates. *Quarately Journal of Economics* 116, 681-704.

Soetevent, A. R. and P. Kooreman (2007). A discrete choice model with social interactions with an application to high school teen behaviors. *Journal of Applied Econometrics* 22, 599-624.

Trogdon, J. G., Nonnemaker, J., and Pais, J. (2008). Peer effects in adolescent overweight. *Journal of Health Economics*, 27, 1388-1399.

Udry, J. R. (2003). The national longitudinal study of adolescent health (add health), waves 1.

Whitmore, D. (2005). Resource and peer impacts on girls' academic achievement: Evidence from a randomized experiment. *American Economic Review*, 95, 199-203.

Wooldridge J. M. 2010. *Econometric Analysis of Cross Section and Panel Data*. MIT Press Books, The MIT Press.

Zeger, S. L. and Karim, M. R. (1991). Generalized linear models with random effects: A Gibbs sampling approach. *Journal of American Statistical Association*, 86, 79-86.

Zimmerman, D. J. (2003). Peer effects in academic outcomes: Evidence from a natural experiment. *Review of Economics and Statistics* 85, 9-23.

Table 1: Summary Statistics

Variable	Min	Max	GPA Sample Mean	GPA Sample S.D.	Smoking Sample Mean	Smoking Sample S.D.
GPA by the following groups:						
Male	1	4	2.778	0.749	-	-
Female	1	4	2.904	0.739	-	-
White	1	4	2.945	0.750	-	-
Black	1	4	2.658	0.688	-	-
Other racial groups	1	4	2.827	0.780	-	-
Smoking by the following groups:						
Male	0	30	-	-	4.307	9.745
Female	0	30	-	-	4.625	10.059
White	0	30	-	-	5.286	10.575
Black	0	30	-	-	1.840	6.448
Other racial groups	0	30	-	-	5.083	10.738
Explanatory variables:						
Male	0	1	0.509	0.500	0.470	0.499
Female	0	1	0.491	0.500	0.530	0.499
Age	13	19	15.508	1.249	15.436	1.215
White	0	1	0.560	0.496	0.648	0.477
Black	0	1	0.313	0.463	0.228	0.420
Asian	0	1	0.021	0.143	0.037	0.189
Hispanic	0	1	0.057	0.232	0.046	0.209
Other race	0	1	0.049	0.216	0.041	0.199
Both parents	0	1	0.705	0.456	0.768	0.422
Less HS	0	1	0.101	0.302	0.085	0.279
HS	0	1	0.341	0.474	0.323	0.468
More HS	0	1	0.407	0.491	0.467	0.499
Edu missing	0	1	0.075	0.263	0.069	0.253
Professional	0	1	0.268	0.442	0.307	0.461
Staying home	0	1	0.217	0.412	0.210	0.408
Other Jobs	0	1	0.346	0.475	0.351	0.477
Job missing	0	1	0.082	0.275	0.069	0.253
Welfare	0	1	0.011	0.105	0.006	0.079
Network properties:						
Network size	110	354	202.928	47.107	240.642	78.251
Outdegree	0	10	3.660	2.891	4.238	2.913
Indegree	0	23	3.660	3.419	4.238	3.525
Sample size			2,841		3,369	
Num. of networks			14		14	

Note:

1. Variable descriptions.

Both parents: living with both parents.

Less HS: mother's education is less than high school.

Edu missing: mother's education level is missing.

Professional: mother's job is either scientist, teacher, executive, director, and the like.

Other jobs: mother's occupation is not among "professional" or "staying home".

Welfare: mother participates in social welfare programs.

2. The variables in italics are the omitted categories in estimation.

Table 2: Average number of nominated friends within and across gender

	Female	Male	Sum
GPA sample:			
Female	2.317	1.589	3.906
Male	1.490	1.930	3.420
Smoking sample:			
Female	2.576	1.960	4.536
Male	1.615	2.332	3.947

Table 3: Average number of nominated friends within and across race

	White	Black	Other	Sum
GPA sample:				
White	3.959	0.087	0.419	4.465
Black	0.145	2.228	0.132	2.505
Other	1.942	0.366	0.654	2.961
Smoking sample:				
White	4.041	0.117	0.449	4.607
Black	0.306	2.896	0.211	3.413
Other	2.481	0.392	0.957	3.830

Table 4: GPA: Gender Peer Effects Without Endogenous Network Formation

	Model (1)	Model (2)	Model (3)	Model (4)
Endogenous effects:				
λ_{ff}	0.065^{***}		0.146^{***}	0.140^{***}
	(0.016)		(0.024)	(0.024)
λ_{fm}	0.009		0.045^{**}	0.025
	(0.015)		(0.020)	(0.020)
λ_{mm}	0.035^{**}		0.133^{***}	0.111^{***}
	(0.016)		(0.025)	(0.024)
λ_{mf}	0.045^{***}		0.085^{***}	0.081^{***}
	(0.015)		(0.020)	(0.021)
Contextual effects:				
Male		−0.045	0.031	0.060
		(0.055)	(0.093)	(0.093)
Age		−0.005	-0.035^{***}	-0.034^{***}
		(0.005)	(0.006)	(0.006)
Black		−0.001	−0.024	0.034
		(0.065)	(0.061)	(0.064)
Asian		0.300^{*}	0.122	0.250
		(0.177)	(0.171)	(0.175)
Hispanic		−0.088	−0.071	−0.087
		(0.115)	(0.112)	(0.113)
Other race		0.052	0.024	0.095
		(0.125)	(0.021)	(0.124)
Both Parents		0.159^{**}	0.110^{*}	0.097
		(0.062)	(0.060)	(0.061)
Less HS		-0.233^{***}	-0.148^{*}	-0.165^{*}
		(0.090)	(0.088)	(0.089)
More HS		0.165^{***}	0.068	0.113^{*}
		(0.061)	(0.055)	(0.061)
Edu missing		−0.158	−0.143	−0.105
		(0.108)	(0.107)	(0.106)
Welfare		-0.623^{***}	−0.396	-0.500^{**}
		(0.248)	(0.244)	(0.244)
Job missing		0.023	0.026	0.032
		(0.110)	(0.108)	(0.109)
Professional		−0.026	−0.072	−0.063
		(0.075)	(0.073)	(0.075)
Other Jobs		−0.013	0.023	−0.013
		(0.067)	(0.064)	(0.067)
Own effects:				
Male	-0.097^{*}	-0.105^{***}	-0.178^{***}	-0.165^{***}
	(0.052)	(0.029)	(0.052)	(0.052)
Age	-0.028^{**}	-0.031^{**}	0.001	−0.016
	(0.013)	(0.013)	(0.011)	(0.014)
Black	-0.141^{***}	-0.130^{**}	-0.162^{***}	-0.130^{**}
	(0.047)	(0.056)	(0.049)	(0.055)
Asian	0.222^{**}	0.182^{*}	0.142	0.184^{*}
	(0.096)	(0.099)	(0.097)	(0.097)
Hispanic	−0.071	−0.071	−0.058	−0.056
	(0.062)	(0.063)	(0.061)	(0.063)
Other race	−0.016	−0.022	−0.008	0.004

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Table – Continued

	(0.063)	(0.064)	(0.063)	(0.063)
Both Parents	0.096***	0.096***	0.097***	0.092***
	(0.032)	(0.032)	(0.031)	(0.031)
Less HS	−0.103**	−0.088*	−0.090*	−0.082*
	(0.047)	(0.047)	(0.047)	(0.047)
More HS	0.184***	0.192***	0.167***	0.179***
	(0.033)	(0.034)	(0.032)	(0.033)
Edu missing	0.057	0.062	0.044	0.054
	(0.053)	(0.054)	(0.053)	(0.053)
Welfare	−0.039	0.003	0.013	−0.005
	(0.128)	(0.129)	(0.127)	(0.127)
Job missing	−0.105**	−0.111**	−0.121**	−0.115**
	(0.053)	(0.054)	(0.053)	(0.053)
Professional	0.069*	0.071*	0.069*	0.065*
	(0.039)	(0.039)	(0.039)	(0.038)
Other Jobs	−0.013	−0.004	0.002	−0.009
	(0.035)	(0.035)	(0.034)	(0.034)
Fixed Effect	Yes	Yes	No	Yes

Note:

1. Model (1): The SAR model without contextual effects.
Model (2): The SAR model without endogenous effects.
Model (3): The SAR model without group fixed effects.
Model (4): The full SAR model.

2. The MCMC runs for 150,000 iterations and the first 20,000 iterations are dropped for burn-in.

3. Standard deviations of the posterior draws are in parentheses.

4. * Significant at the 10% level. ** Significant at the 5% level. *** Significant at the 1% level. The indication of significance is based on frequentist's perspective.

Table 5: GPA: Gender Peer Effects With Endogenous Network Formation

	Model (5)	Model (6)	Model (7)	Model (8)
Endogenous effects:				
λ_{ff}	0.136***	0.136***	0.109***	0.102***
	(0.023)	(0.024)	(0.024)	(0.022)
λ_{fm}	0.025	0.019	0.013	0.002
	(0.020)	(0.020)	(0.020)	(0.019)
λ_{mm}	0.109***	0.101***	0.077***	0.061**
	(0.025)	(0.024)	(0.025)	(0.024)
λ_{mf}	0.078***	0.079***	0.067***	0.060***
	(0.020)	(0.021)	(0.020)	(0.020)
Contextual effects:				
Male	0.058	0.055	0.027	0.055
	(0.093)	(0.092)	(0.091)	(0.089)
Age	−0.033***	−0.031***	−0.024***	−0.024***
	(0.006)	(0.006)	(0.007)	(0.006)
Black	0.008	−0.006	0.028	0.112*
	(0.064)	(0.065)	(0.064)	(0.066)
Asian	0.256	0.201	0.112	0.141
	(0.175)	(0.177)	(0.174)	(0.176)
Hispanic	−0.079	−0.064	−0.031	−0.024
	(0.113)	(0.114)	(0.113)	(0.113)
Other race	0.093	0.090	0.096	0.123
	(0.123)	(0.124)	(0.121)	(0.121)
Both Parents	0.102*	0.093	0.087	0.058
	(0.061)	(0.061)	(0.062)	(0.061)
Less HS	−0.167*	−0.161*	−0.173**	−0.105
	(0.089)	(0.088)	(0.088)	(0.087)
More HS	0.111*	0.087	0.047	0.048
	(0.061)	(0.061)	(0.061)	(0.060)
Edu missing	−0.089	−0.085	−0.068	−0.033
	(0.107)	(0.107)	(0.107)	(0.106)
Welfare	−0.480*	−0.479*	−0.504**	−0.507**
	(0.246)	(0.247)	(0.249)	(0.248)
Job missing	0.020	0.041	0.021	0.008
	(0.109)	(0.109)	(0.107)	(0.108)
Professional	−0.058	−0.051	−0.058	−0.065
	(0.075)	(0.074)	(0.075)	(0.073)
Other Jobs	−0.001	0.001	−0.025	−0.026
	(0.067)	(0.067)	(0.067)	(0.066)
Own effects:				
Male	−0.166***	−0.168***	−0.174***	−0.166***
	(0.051)	(0.053)	(0.052)	(0.053)
Age	−0.017	−0.015	−0.018	−0.017
	(0.011)	(0.011)	(0.012)	(0.012)
Black	−0.151***	−0.157***	−0.130*	−0.072
	(0.056)	(0.056)	(0.055)	(0.056)
Asian	0.173*	0.157	0.125	0.126
	(0.098)	(0.097)	(0.095)	(0.095)
Hispanic	−0.058	−0.059	−0.054	−0.042
	(0.062)	(0.062)	(0.061)	(0.061)
Other race	0.002	0.006	0.015	0.015

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Table – Continued

	(0.063)	(0.063)	(0.062)	(0.061)
Both Parents	0.097***	0.087***	0.081***	0.070**
	(0.031)	(0.031)	(0.031)	(0.031)
Less HS	−0.084*	−0.078*	−0.077*	−0.068
	(0.047)	(0.046)	(0.046)	(0.045)
More HS	0.178***	0.174***	0.162***	0.159***
	(0.033)	(0.033)	(0.033)	(0.032)
Edu missing	0.054	0.054	0.051	0.058
	(0.053)	(0.053)	(0.053)	(0.052)
Welfare	−0.004	−0.002	0.002	−0.006
	(0.127)	(0.127)	(0.127)	(0.127)
Job missing	−0.120**	−0.108**	−0.102**	−0.099*
	(0.053)	(0.053)	(0.052)	(0.052)
Professional	0.065*	0.070*	0.064*	0.059
	(0.038)	(0.038)	(0.038)	(0.037)
Other Jobs	−0.009	−0.006	−0.012	−0.019
	(0.034)	(0.034)	(0.033)	(0.033)
Network variables:				
Constant	−3.762***	−2.244***	−0.905***	0.388***
	(0.041)	(0.043)	(0.054)	(0.062)
Same Grade	2.021***	2.128***	2.197***	2.292***
	(0.025)	(0.027)	(0.032)	(0.036)
Same Sex	0.327***	0.294***	0.291***	0.271***
	(0.022)	(0.025)	(0.026)	(0.029)
Same Race	0.550***	0.521***	0.553***	0.598***
	(0.032)	(0.036)	(0.040)	(0.045)
δ_1	−4.461***	−3.228***	−2.746***	−2.705***
	(0.095)	(0.072)	(0.077)	(0.084)
δ_2	–	−3.020***	−2.567***	−2.334***
	–	(0.066)	(0.056)	(0.068)
δ_3	–	–	−2.445***	−2.185***
	–	–	(0.059)	(0.060)
δ_4	–	–	–	−2.033***
	–	–	–	(0.063)
Fixed Effect	Yes	Yes	Yes	Yes
AICM	-94,510	-86,299	-82,936	-84,326
S.E.(AICM)	134.048	149.767	177.542	227.103

Note:

1. Model (5): The SCSAR model with unobservable Z in one dimension.
Model (6): The SCSAR model with unobservable Z in two dimensions.
Model (7): The SCSAR model with unobservable Z in three dimensions.
Model (8): The SCSAR model with unobservable Z in four dimensions.
2. The MCMC runs for 150,000 iterations and the first 20,000 iterations are dropped for burn-in.
3. Standard deviations of the posterior draws are in parentheses.
4. * Significant at the 10% level. ** Significant at the 5% level.
*** Significant at the 1% level. The indication of significance is based on frequentist's perspective.

Table 6: Smoking: Gender Peer Effects Without Endogenous Network Formation

	Model (1)	Model (2)	Model (3)	Model (4)
Endogenous effects:				
λ_{ff}	0.463***		0.489***	0.479***
	(0.031)		(0.031)	(0.031)
λ_{fm}	0.161***		0.213***	0.198***
	(0.034)		(0.033)	(0.034)
λ_{mm}	0.262***		0.334***	0.320***
	(0.038)		(0.037)	(0.038)
λ_{mf}	0.197***		0.246***	0.233***
	(0.037)		(0.037)	(0.037)
Contextual effects:				
Male		−1.278*	−0.922	−0.942
		(0.667)	(0.641)	(0.644)
Age		−0.048	−0.316***	−0.318***
		(0.065)	(0.062)	(0.063)
Black		−2.165***	0.203	0.211
		(0.816)	(0.747)	(0.774)
Asian		−4.211***	−3.364**	−3.321**
		(1.575)	(1.437)	(1.507)
Hispanic		1.399	0.786	1.415
		(1.398)	(1.306)	(1.331)
Other race		3.700**	1.635	2.243
		(1.512)	(1.405)	(1.440)
Both Parents		−1.731**	0.334	0.129
		(0.773)	(0.726)	(0.734)
Less HS		3.839***	2.078*	2.105*
		(1.135)	(1.063)	(1.079)
More HS		−0.373	−0.096	0.023
		(0.737)	(0.641)	(0.698)
Edu missing		1.203	0.347	0.666
		(1.295)	(1.221)	(1.230)
Welfare		0.051	−0.641)	−0.678
		(2.511)	(2.451	(2.461)
Job missing		3.964***	3.914***	4.333***
		(1.277)	(1.196)	(1.205)
Professional		−0.283	−0.153	−0.106
		(0.871)	(0.799)	(0.820)
Other Jobs		0.455	−0.017	0.051
		(0.808)	(0.724)	(0.767)
Own effects:				
Male	0.296	−0.467	0.039	0.065
	(0.363)	(0.362)	(0.381)	(0.383)
Age	0.728***	0.985***	0.761***	0.769***
	(0.123)	(0.136)	(0.114)	(0.136)
Black	−1.999***	−2.441***	−2.551***	−2.226***
	(0.519)	(0.685)	(0.612)	(0.644)
Asian	0.344	0.050	0.612	0.707
	(0.858)	(0.919)	(0.859)	(0.867)
Hispanic	0.196	−0.838	−0.808	−0.434
	(0.764)	(0.811)	(0.758)	(0.766)
Other race	1.998**	1.760**	1.567**	1.671**

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Table – Continued

	(0.787)	(0.820)	(0.769)	(0.774)
Both Parents	-2.162^{***}	-2.392^{***}	-1.927^{***}	-1.995^{***}
	(0.392)	(0.413)	(0.387)	(0.389)
Less HS	1.528^{***}	1.300^{**}	1.244^{**}	1.294^{**}
	(0.594)	(0.626)	(0.587)	(0.587)
More HS	-0.094	-0.181	0.041	0.091
	(0.397)	(0.417)	(0.379)	(0.392)
Edu missing	0.434	-0.286	-0.010	0.008
	(0.652)	(0.684)	(0.641)	(0.641)
Welfare	3.314^{*}	3.761^{**}	3.468^{**}	3.283^{*}
	(1.712)	(1.775)	(1.695)	(1.696)
Job missing	0.304	0.504	-0.248	-0.057
	(0.673)	(0.700)	(0.658)	(0.659)
Professional	-0.586	-0.333	-0.447	-0.432
	(0.453)	(0.473)	(0.443)	(0.445)
Other Jobs	0.405	0.760	0.541	0.576
	(0.412)	(0.432)	(0.397)	(0.406)
Fixed Effect	Yes	Yes	No	Yes

Note:

1. Model (1): The SAR model without contextual effects.
Model (2): The SAR model without endogenous effects.
Model (3): The SAR model without group fixed effects.
Model (4): The full SAR model.

2. The MCMC runs for 150,000 iterations and the first 20,000 iterations are dropped for burn-in.

3. Standard deviations of the posterior draws are in parentheses.

4. * Significant at the 10% level. ** Significant at the 5% level. *** Significant at the 1% level. The indication of significance is based on frequentist's perspective.

Table 7: Smoking: Gender Peer Effects With Endogenous Network Formation

	Model (5)	Model (6)	Model (7)	Model (8)
Endogenous effects:				
λ_{ff}	0.476***	0.476***	0.477***	0.487***
	(0.031)	(0.032)	(0.031)	(0.031)
λ_{fm}	0.198***	0.198***	0.198***	0.203***
	(0.034)	(0.034)	(0.034)	(0.034)
λ_{mm}	0.322***	0.320***	0.318***	0.325***
	(0.038)	(0.038)	(0.038)	(0.038)
λ_{mf}	0.232***	0.232***	0.231***	0.238***
	(0.037)	(0.037)	(0.037)	(0.037)
Contextual effects:				
Male	−1.005	−0.966	−0.992	−0.971
	(0.661)	(0.663)	(0.662)	(0.647)
Age	−0.334***	−0.328***	−0.332***	−0.326***
	(0.065)	(0.065)	(0.065)	(0.064)
Black	0.265	0.180	0.213	0.321
	(0.815)	(0.820)	(0.820)	(0.790)
Asian	−4.183**	−4.007**	−3.907**	−3.175**
	(1.723)	(1.724)	(1.722)	(1.516)
Hispanic	1.586	1.578	1.713	1.415
	(1.481)	(1.483)	(1.483)	(1.332)
Other race	2.818*	2.986*	2.684*	1.968
	(1.628)	(1.625)	(1.628)	(1.438)
Both Parents	0.159	0.083	0.129	0.143
	(0.762)	(0.761)	(0.764)	(0.738)
Less HS	2.315	2.373	2.516	1.879
	(1.152)	(1.157)	(1.158)	(1.074)
More HS	0.148	0.075	−0.053	0.259
	(0.727)	(0.726)	(0.730)	(0.705)
Edu missing	0.735	0.683	0.723	0.760
	(1.351)	(1.347)	(1.348)	(1.231)
Welfare	−1.876	−2.134	−1.950	−0.419
	(3.935)	(3.957)	(3.939)	(2.461)
Job missing	5.079***	5.224***	5.146***	4.261***
	(1.330)	(1.335)	(1.333)	(1.213)
Professional	0.102	0.058	0.205	0.003
	(0.869)	(0.863)	(0.869)	(0.823)
Other Jobs	0.237	0.229	0.348	−0.035
	(0.809)	(0.809)	(0.809)	(0.771)
Own effects:				
Male	0.064	0.054	0.061	0.084
	(0.386)	(0.387)	(0.386)	(0.378)
Age	0.772***	0.783***	0.776***	0.728***
	(0.134)	(0.129)	(0.131)	(0.138)
Black	−2.323***	−2.300***	−2.406***	−2.163***
	(0.669)	(0.674)	(0.672)	(0.650)
Asian	0.874	0.825	0.918	0.770
	(0.915)	(0.913)	(0.916)	(0.868)
Hispanic	−0.512	−0.425	−0.469	−0.352
	(0.792)	(0.795)	(0.792)	(0.764)
Other race	1.720**	1.731**	1.735**	1.656**

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Table – Continued

	(0.802)	(0.802)	(0.800)	(0.771)
Both Parents	-2.028^{***}	-2.021^{***}	-2.015^{***}	-1.950^{***}
	(0.393)	(0.394)	(0.394)	(0.391)
Less HS	1.284^{**}	1.274^{**}	1.352^{**}	1.247^{**}
	(0.600)	(0.601)	(0.601)	(0.585)
More HS	0.095	0.075	0.051	0.158
	(0.396)	(0.397)	(0.398)	(0.391)
Edu missing	−0.050	−0.025	−0.037	0.012
	(0.661)	(0.660)	(0.659)	(0.640)
Welfare	4.712^{**}	4.741^{**}	4.856^{**}	3.143^{*}
	(2.022)	(2.026)	(2.027)	(1.692)
Job missing	−0.028	0.043	0.018	−0.108
	(0.682)	(0.680)	(0.682)	(0.656)
Professional	−0.385	−0.381	−0.369	−0.430
	(0.453)	(0.455)	(0.453)	(0.444)
Other Jobs	0.631	0.643	0.641	0.516
	(0.414)	(0.415)	(0.413)	(0.407)
Network variables:				
Constant	-4.106^{***}	-2.589^{***}	-1.236^{***}	0.033^{***}
	(0.033)	(0.037)	(0.043)	(0.055)
Same Grade	2.166^{***}	2.246^{***}	2.312^{***}	2.388^{***}
	(0.022)	(0.024)	(0.027)	(0.022)
Same Sex	0.326^{***}	0.300^{***}	0.274^{***}	0.244^{***}
	(0.019)	(0.021)	(0.022)	(0.023)
Same Race	0.681^{***}	0.630^{***}	0.650^{***}	0.706^{***}
	(0.027)	(0.029)	(0.033)	(0.041)
δ_1	-3.692^{***}	-2.825^{***}	-2.574^{***}	-2.394^{***}
	(0.074)	(0.044)	(0.049)	(0.067)
δ_2	–	-2.776^{***}	-2.472^{***}	-2.263^{***}
	–	(0.042)	(0.045)	(0.043)
δ_3	–	–	-2.155^{***}	-2.028^{***}
	–	–	(0.051)	(0.044)
δ_4	–	–	–	-1.975^{***}
	–	–	–	(0.038)
Fixed Effect	Yes	Yes	Yes	Yes
AICM	-142,340	-129,510	-127,940	-132,290
S.E.(AICM)	123.641	140.675	218.785	329.065

Note:

1. Model (5): The SCSAR model with unobservable Z in one dimension.
Model (6): The SCSAR model with unobservable Z in two dimensions.
Model (7): The SCSAR model with unobservable Z in three dimensions.
Model (8): The SCSAR model with unobservable Z in four dimensions.
2. The MCMC runs for 150,000 iterations and the first 20,000 iterations are dropped for burn-in.
3. Standard deviations of the posterior draws are in parentheses.
4. * Significant at the 10% level. ** Significant at the 5% level.
*** Significant at the 1% level. The indication of significance is based on frequentist's perspective.

Table 8: GPA: Racial Peer Effects Without Endogenous Network Formation

	Model (1)	Model (2)	Model (3)	Model (4)
Endogenous effects:				
λ_{ww}	0.100***		0.281***	0.253***
	(0.016)		(0.030)	(0.028)
λ_{wb}	−0.014		−0.035	−0.025
	(0.026)		(0.026)	(0.026)
λ_{wo}	−0.005		−0.043**	−0.036**
	(0.013)		(0.017)	(0.017)
λ_{bb}	0.037*		0.120***	0.112***
	(0.019)		(0.040)	(0.040)
λ_{bw}	0.026		0.122***	0.124***
	(0.034)		(0.036)	(0.037)
λ_{bo}	0.036		0.035	0.036
	(0.029)		(0.032)	(0.033)
λ_{oo}	0.028		0.021	0.022
	(0.027)		(0.032)	(0.032)
λ_{ow}	0.050*		0.210***	0.189***
	(0.027)		(0.039)	(0.034)
λ_{ob}	−0.028		−0.022	−0.014
	(0.036)		(0.041)	(0.040)
Contextual effects:				
Male		−0.046	−0.014	−0.023
		(0.055)	(0.054)	(0.054)
Age		−0.005	−0.049***	−0.046***
		(0.005)	(0.007)	(0.007)
Black		−0.001	0.408***	0.409***
		(0.065)	(0.139)	(0.137)
Asian		0.300*	0.584***	0.658***
		(0.177)	(0.204)	(0.206)
Hispanic		−0.088	0.478***	0.405***
		(0.115)	(0.159)	(0.155)
Other race		0.052	0.492***	0.513***
		(0.125)	(0.160)	(0.158)
Both Parents		0.158**	0.126**	0.104**
		(0.062)	(0.060)	(0.061)
Less HS		−0.233***	−0.201**	−0.211**
		(0.090)	(0.087)	(0.088)
More HS		0.165***	0.053	0.093
		(0.061)	(0.055)	(0.060)
Edu missing		−0.158	−0.125	−0.075
		(0.108)	(0.107)	(0.107)
Welfare		−0.623**	−0.552**	−0.639***
		(0.248)	(0.243)	(0.244)
Job missing		0.022	0.037	0.043
		(0.110)	(0.107)	(0.109)
Professional		−0.026	−0.091	−0.077
		(0.075)	(0.073)	(0.074)
Other Jobs		−0.013	−0.018	−0.036
		(0.067)	(0.063)	(0.067)
Own effects:				
Male	−0.098***	−0.105***	−0.128***	−0.116***

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Table – Continued

	(0.027)	(0.029)	(0.029)	(0.029)
Age	-0.031^{**}	-0.032^{***}	0.005	-0.014
	(0.012)	(0.012)	(0.011)	(0.013)
Black	-0.063	-0.130^{**}	-0.172^{***}	-0.151^{**}
	(0.070)	(0.056)	(0.060)	(0.070)
Asian	0.319^{***}	0.182^{*}	0.206^{*}	0.243^{**}
	(0.119)	(0.099)	(0.118)	(0.119)
Hispanic	0.018	-0.071	0.009	0.008
	(0.083)	(0.063)	(0.081)	(0.083)
Other race	0.078	-0.022	0.069	0.079
	(0.091)	(0.064)	(0.089)	(0.090)
Both Parents	0.094^{***}	0.096^{***}	0.094^{***}	0.087^{***}
	(0.032)	(0.032)	(0.031)	(0.031)
Less HS	-0.093^{**}	-0.088^{*}	-0.083^{*}	-0.077^{*}
	(0.047)	(0.047)	(0.046)	(0.046)
More HS	0.186^{***}	0.192^{***}	0.173^{***}	0.183^{***}
	(0.033)	(0.034)	(0.032)	(0.033)
Edu missing	0.060	0.062	0.038	0.050
	(0.054)	(0.054)	(0.053)	(0.053)
Welfare	-0.030	0.003	0.076	0.053
	(0.127)	(0.129)	(0.128)	(0.126)
Job missing	-0.104^{*}	-0.111^{**}	-0.121^{**}	-0.112^{**}
	(0.053)	(0.054)	(0.053)	(0.053)
Professional	0.071^{*}	0.071^{*}	0.077^{**}	0.074^{*}
	(0.039)	(0.039)	(0.038)	(0.038)
Other Jobs	-0.015	-0.004	0.002	-0.007
	(0.034)	(0.035)	(0.034)	(0.034)
Fixed Effect	Yes	Yes	No	Yes

Note:

1. Model (1): The SAR model without contextual effects.
Model (2): The SAR model without endogenous effects.
Model (3): The SAR model without group fixed effects.
Model (4): The full SAR model.

2. The MCMC runs for 150,000 iterations and the first 20,000 iterations are dropped for burn-in.

3. Standard deviations of the posterior draws are in parentheses.

4. * Significant at the 10% level. ** Significant at the 5% level. *** Significant at the 1% level. The indication of significance is based on frequentist's perspective.

Table 9: GPA: Racial Peer Effects With Endogenous Network Formation

	Model (5)	Model (6)	Model (7)	Model (8)
Endogenous effects:				
λ_{ww}	0.255***	0.243***	0.194***	0.175***
	(0.029)	(0.026)	(0.028)	(0.029)
λ_{wb}	−0.027	−0.029	−0.025	−0.033
	(0.027)	(0.027)	(0.027)	(0.027)
λ_{wo}	−0.036**	−0.034**	−0.033**	−0.029**
	(0.018)	(0.018)	(0.017)	(0.017)
λ_{bb}	0.106***	0.105**	0.079*	0.058
	(0.040)	(0.042)	(0.042)	(0.039)
λ_{bw}	0.118***	0.120***	0.098***	0.087**
	(0.037)	(0.037)	(0.036)	(0.037)
λ_{bo}	0.039	0.032	0.013	0.016
	(0.034)	(0.033)	(0.033)	(0.033)
λ_{oo}	0.021	0.019	0.010	0.006
	(0.030)	(0.029)	(0.029)	(0.030)
λ_{ow}	0.189**	0.180**	0.139**	0.141**
	(0.036)	(0.034)	(0.035)	(0.037)
λ_{ob}	−0.014	−0.024	−0.029	−0.027
	(0.040)	(0.040)	(0.041)	(0.039)
Contextual effects:				
Male	−0.021	−0.024	−0.046	−0.053
	(0.054)	(0.054)	(0.054)	(0.054)
Age	−0.046***	−0.043***	−0.035***	−0.034***
	(0.007)	(0.007)	(0.007)	(0.007)
Black	0.441***	0.376***	0.350**	0.395***
	(0.144)	(0.138)	(0.144)	(0.140)
Asian	0.627***	0.620***	0.560***	0.410*
	(0.207)	(0.204)	(0.209)	(0.216)
Hispanic	0.393**	0.416***	0.405***	0.385**
	(0.158)	(0.153)	(0.153)	(0.155)
Other race	0.501***	0.488***	0.494***	0.445***
	(0.159)	(0.157)	(0.155)	(0.155)
Both Parents	0.091	0.099	0.088	0.065
	(0.061)	(0.062)	(0.061)	(0.061)
Less HS	−0.193**	−0.228***	−0.205**	−0.167*
	(0.088)	(0.088)	(0.089)	(0.088)
More HS	0.093	0.098	0.058	0.062
	(0.060)	(0.061)	(0.060)	(0.060)
Edu missing	−0.073	−0.070	−0.019	−0.007
	(0.107)	(0.107)	(0.105)	(0.108)
Welfare	−0.660***	−0.643***	−0.702***	−0.642***
	(0.243)	(0.245)	(0.249)	(0.251)
Job missing	0.042	0.036	0.029	0.083
	(0.109)	(0.108)	(0.108)	(0.110)
Professional	−0.066	−0.080	−0.075	−0.048
	(0.074)	(0.074)	(0.073)	(0.074)
Other Jobs	−0.032	−0.036	−0.008	−0.002
	(0.066)	(0.066)	(0.067)	(0.068)
Own effects:				
Male	−0.115***	−0.118***	−0.124***	−0.129***

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Table – Continued

	(0.029)	(0.029)	(0.028)	(0.028)
Age	−0.016	−0.016	−0.010	−0.021
	(0.012)	(0.012)	(0.012)	(0.013)
Black	−0.127*	−0.179**	−0.146**	−0.121*
	(0.070)	(0.071)	(0.074)	(0.074)
Asian	0.244**	0.240**	0.200*	0.141
	(0.119)	(0.116)	(0.118)	(0.120)
Hispanic	0.010	0.010	0.019	−0.011
	(0.084)	(0.081)	(0.083)	(0.085)
Other race	0.076	0.081	0.098	0.052
	(0.090)	(0.088)	(0.090)	(0.090)
Both Parents	0.085***	0.085***	0.077***	0.071***
	(0.031)	(0.031)	(0.031)	(0.031)
Less HS	−0.076*	−0.080*	−0.078*	−0.067
	(0.046)	(0.046)	(0.046)	(0.045)
More HS	0.182***	0.182***	0.173***	0.162***
	(0.033)	(0.033)	(0.032)	(0.032)
Edu missing	0.047	0.054	0.054	0.050
	(0.053)	(0.053)	(0.052)	(0.052)
Welfare	0.040	0.046	0.025	0.030
	(0.126)	(0.127)	(0.128)	(0.126)
Job missing	−0.112**	−0.114**	−0.111**	−0.098**
	(0.052)	(0.052)	(0.052)	(0.052)
Professional	0.073*	0.074*	0.067*	0.073*
	(0.038)	(0.038)	(0.037)	(0.037)
Other Jobs	−0.009	−0.008	−0.007	−0.007
	(0.034)	(0.034)	(0.033)	(0.033)
Network variables:				
Constant	−3.749***	−2.242***	−0.887***	0.403***
	(0.039)	(0.045)	(0.052)	(0.066)
Same Grade	2.012***	2.115***	2.209***	2.272***
	(0.026)	(0.028)	(0.032)	(0.036)
Same Sex	0.318***	0.298***	0.284***	0.267***
	(0.022)	(0.024)	(0.028)	(0.029)
Same Race	0.559***	0.523***	0.536***	0.598***
	(0.032)	(0.035)	(0.040)	(0.046)
δ_1	−4.508***	−3.152***	−2.683***	−2.611***
	(0.099)	(0.055)	(0.058)	(0.079)
δ_2	–	−3.079***	−2.606***	−2.373***
	–	(0.054)	(0.047)	(0.069)
δ_3	–	–	−2.488***	−2.190***
	–	–	(0.061)	(0.059)
δ_4	–	–	–	−2.085***
	–	–	–	(0.057)
Fixed Effect	Yes	Yes	Yes	Yes
AICM	-90,553	-81,855	-81,535	-84,303
S.E.(AICM)	89.261	132.852	165.140	212.593

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Table – Continued

Note:

1. Model (5): The SCSAR model with unobservable Z in one dimension.
Model (6): The SCSAR model with unobservable Z in two dimensions.
Model (7): The SCSAR model with unobservable Z in three dimensions.
Model (8): The SCSAR model with unobservable Z in four dimensions.

2. The MCMC runs for 150,000 iterations and the first 20,000 iterations are dropped for burn-in.

3. Standard deviations of the posterior draws are in parentheses.

4. * Significant at the 10% level. ** Significant at the 5% level.
*** Significant at the 1% level. The indication of significance is based on frequentist's perspective.

Table 10: Smoking: Racial Peer Effects Without Endogenous Network Formation

	Model (1)	Model (2)	Model (3)	Model (4)
Endogenous effects:				
λ_{ww}	0.466***		0.527***	0.507***
	(0.029)		(0.029)	(0.029)
λ_{wb}	−0.035		−0.015	−0.019
	(0.058)		(0.057)	(0.056)
λ_{wo}	0.114***		0.129***	0.126***
	(0.030)		(0.030)	(0.030)
λ_{bb}	−0.023		0.117	0.078
	(0.102)		(0.101)	(0.102)
λ_{bw}	0.338***		0.399***	0.372***
	(0.085)		(0.086)	(0.087)
λ_{bo}	0.152		0.225	0.203
	(0.175)		(0.171)	(0.169)
λ_{oo}	0.316***		0.333***	0.330***
	(0.073)		(0.073)	(0.073)
λ_{ow}	0.333***		0.376***	0.365***
	(0.071)		(0.070)	(0.070)
λ_{ob}	−0.232**		−0.164	−0.185*
	(0.114)		(0.113)	(0.110)
Contextual effects:				
Male		−1.264*	−0.825	−0.916
		(0.667)	(0.631)	(0.635)
Age		−0.049	−0.288***	−0.301***
		(0.065)	(0.061)	(0.063)
Black		−2.142***	0.146	0.119
		(0.814)	(0.758)	(0.787)
Asian		−4.214***	−3.875***	−3.782**
		(1.575)	(1.447)	(1.516)
Hispanic		1.417	0.952	1.625
		(1.404)	(1.319)	(1.347)
Other race		3.698**	1.149	1.806
		(1.510)	(1.452)	(1.480)
Both Parents		−1.733**	0.188	−0.013
		(0.774)	(0.732)	(0.741)
Less HS		3.850***	1.818*	1.739
		(1.130)	(1.069)	(1.081)
More HS		−0.341	−0.153	0.106
		(0.738)	(0.644)	(0.696)
Edu missing		1.180	0.494	0.947
		(1.296)	(1.225)	(1.234)
Welfare		0.018	0.430	0.235
		(2.515)	(2.457)	(2.465)
Job missing		3.950***	3.116**	3.630***
		(1.273)	(1.192)	(1.210)
Professional		−0.311	−0.374	−0.286
		(0.868)	(0.800)	(0.821)
Other Jobs		0.462	−0.199	0.013
		(0.806)	(0.722)	(0.765)
Own effects:				
Male	−0.181	−0.464	−0.304	−0.302

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Table – Continued

	(0.321)	(0.360)	(0.341)	(0.341)
Age	0.714***	0.990***	0.783***	0.786***
	(0.131)	(0.142)	(0.115)	(0.131)
Black	−1.818***	−2.459***	−2.493***	−2.091***
	(0.544)	(0.685)	(0.635)	(0.673)
Asian	0.371	0.047	0.666	0.777
	(0.874)	(0.918)	(0.872)	(0.885)
Hispanic	0.198	−0.848	−0.864	−0.398
	(0.794)	(0.809)	(0.787)	(0.799)
Other race	1.920**	1.771**	1.450*	1.592*
	(0.847)	(0.822)	(0.832)	(0.839)
Both Parents	−2.344***	−2.402***	−2.064***	−2.134***
	(0.394)	(0.413)	(0.389)	(0.389)
Less HS	1.353**	1.307**	1.121*	1.138*
	(0.597)	(0.625)	(0.587)	(0.590)
More HS	−0.003	−0.184	0.107	0.155
	(0.402)	(0.415)	(0.379)	(0.395)
Edu missing	0.404	−0.300	−0.025	−0.012
	(0.656)	(0.681)	(0.645)	(0.645)
Welfare	3.868**	3.750**	3.901**	3.690**
	(1.719)	(1.777)	(1.700)	(1.704)
Job missing	0.690	0.518	0.117	0.330
	(0.675)	(0.699)	(0.662)	(0.667)
Professional	−0.520	−0.326	−0.370	−0.319
	(0.455)	(0.473)	(0.444)	(0.445)
Other Jobs	0.534	0.768*	0.621	0.717*
	(0.415)	(0.433)	(0.399)	(0.410)
Fixed Effect	Yes	Yes	No	Yes

Note:

1. Model (1): The SAR model without contextual effects.
Model (2): The SAR model without endogenous effects.
Model (3): The SAR model without group fixed effects.
Model (4): The full SAR model.

2. The MCMC runs for 150,000 iterations and the first 20,000 iterations are dropped for burn-in.

3. Standard deviations of the posterior draws are in parentheses.

4. * Significant at the 10% level. ** Significant at the 5% level. *** Significant at the 1% level. The indication of significance is based on frequentist's perspective.

Table 11: Smoking: Racial Peer Effects With Endogenous Network Formation

	Model (5)	Model (6)	Model (7)	Model (8)
Endogenous effects:				
λ_{ww}	0.508***	0.513***	0.511***	0.520***
	(0.029)	(0.028)	(0.029)	(0.029)
λ_{wb}	−0.017	−0.024	−0.028	−0.012
	(0.056)	(0.057)	(0.056)	(0.057)
λ_{wo}	0.124***	0.128***	0.127***	0.134***
	(0.030)	(0.030)	(0.030)	(0.031)
λ_{bb}	0.069	0.083	0.097	0.055
	(0.102)	(0.100)	(0.100)	(0.100)
λ_{bw}	0.364***	0.385***	0.382***	0.378***
	(0.085)	(0.085)	(0.085)	(0.086)
λ_{bo}	0.196	0.206	0.176	0.160
	(0.171)	(0.172)	(0.172)	(0.170)
λ_{oo}	0.328***	0.332***	0.321***	0.337***
	(0.072)	(0.073)	(0.072)	(0.072)
λ_{ow}	0.360***	0.380***	0.370***	0.373***
	(0.069)	(0.069)	(0.072)	(0.070)
λ_{ob}	−0.177	−0.173	−0.164	−0.175
	(0.109)	(0.108)	(0.110)	(0.111)
Contextual effects:				
Male	−0.922	−0.869	−1.016	−0.872
	(0.634)	(0.630)	(0.633)	(0.632)
Age	−0.295***	−0.295***	−0.288***	−0.308***
	(0.063)	(0.063)	(0.063)	(0.063)
Black	0.156	−0.172	−0.037	0.260
	(0.789)	(0.797)	(0.799)	(0.794)
Asian	−3.837**	−3.217**	−3.911***	−3.374**
	(1.511)	(1.509)	(1.522)	(1.527)
Hispanic	1.762	1.803	1.539	1.178
	(1.346)	(1.352)	(1.350)	(1.345)
Other race	1.730	2.017	1.707	1.593
	(1.485)	(1.480)	(1.480)	(1.485)
Both Parents	−0.101	0.089	0.001	0.016
	(0.742)	(0.737)	(0.742)	(0.742)
Less HS	1.786*	1.506	1.251	1.558
	(1.083)	(1.085)	(1.089)	(1.081)
More HS	0.099	0.018	0.212	0.179
	(0.698)	(0.697)	(0.697)	(0.705)
Edu missing	1.082	0.997	0.968	0.877
	(1.237)	(1.233)	(1.238)	(1.236)
Welfare	0.308	0.105	0.192	0.473
	(2.460)	(2.476)	(2.472)	(2.476)
Job missing	3.620***	3.597***	3.771***	3.881***
	(1.217)	(1.209)	(1.223)	(1.214)
Professional	−0.331	−0.479	−0.646	−0.313
	(0.819)	(0.820)	(0.828)	(0.824)
Other Jobs	−0.003	−0.102	−0.165	−0.053
	(0.765)	(0.766)	(0.764)	(0.770)
Own effects:				
Male	−0.289	−0.302	−0.309	−0.320

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Table – Continued

	(0.341)	(0.340)	(0.339)	(0.340)
Age	0.773***	0.765***	0.734***	0.747***
	(0.138)	(0.132)	(0.137)	(0.134)
Black	−2.075***	−2.264***	−2.093***	−1.885***
	(0.670)	(0.674)	(0.679)	(0.678)
Asian	0.838	0.936	0.817	0.883
	(0.879)	(0.879)	(0.882)	(0.886)
Hispanic	−0.337	−0.364	−0.400	−0.443
	(0.797)	(0.797)	(0.798)	(0.796)
Other race	1.599*	1.649**	1.634**	1.586*
	(0.833)	(0.832)	(0.836)	(0.834)
Both Parents	−2.161***	−2.040***	−2.071***	−2.064***
	(0.388)	(0.388)	(0.389)	(0.388)
Less HS	1.144*	1.041*	1.074*	1.109*
	(0.590)	(0.593)	(0.587)	(0.587)
More HS	0.163	0.165	0.232	0.188
	(0.394)	(0.393)	(0.390)	(0.393)
Edu missing	0.055	−0.009	0.041	0.026
	(0.644)	(0.645)	(0.644)	(0.643)
Welfare	3.692**	3.611**	3.723**	3.657**
	(1.701)	(1.699)	(1.703)	(1.700)
Job missing	0.328	0.236	0.356	0.377
	(0.664)	(0.663)	(0.662)	(0.667)
Professional	−0.324	−0.371	−0.382	−0.307
	(0.448)	(0.447)	(0.444)	(0.446)
Other Jobs	0.712*	0.640	0.634	0.704*
	(0.409)	(0.407)	(0.406)	(0.408)
Network variables:				
Constant	−4.175***	−2.576***	−1.258***	0.031***
	(0.035)	(0.039)	(0.043)	(0.050)
Same Grade	2.161***	2.257***	2.339***	2.401***
	(0.022)	(0.024)	(0.027)	(0.028)
Same Sex	0.323***	0.298***	0.266***	0.252***
	(0.019)	(0.021)	(0.022)	(0.023)
Same Race	0.746***	0.622***	0.664***	0.705***
	(0.027)	(0.031)	(0.033)	(0.036)
δ_1	−3.612***	−2.887***	−2.480***	−2.362***
	(0.076)	(0.056)	(0.043)	(0.055)
δ_2	–	−2.752***	−2.387***	−2.230***
	–	(0.049)	(0.036)	(0.047)
δ_3	–	–	−2.311***	−2.114***
	–	–	(0.038)	(0.041)
δ_4	–	–	–	−1.963***
	–	–	–	(0.047)
Fixed Effect	Yes	Yes	Yes	Yes
AICM	-154,830	-129,820	-125,690	-126,680
S.E.(AICM)	244.372	146.665	196.969	274.037

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Table – Continued

Note:

1. Model (5): The SCSAR model with unobservable Z in one dimension.
Model (6): The SCSAR model with unobservable Z in two dimensions.
Model (7): The SCSAR model with unobservable Z in three dimensions.
Model (8): The SCSAR model with unobservable Z in four dimensions.

2. The MCMC runs for 150,000 iterations and the first 20,000 iterations are dropped for burn-in.

3. Standard deviations of the posterior draws are in parentheses.

4. * Significant at the 10% level. ** Significant at the 5% level.
*** Significant at the 1% level. The indication of significance is based on frequentist's perspective.