Information Management in Banking Crises*

Joel Shapiro[†] and David Skeie[‡]
University of Oxford and Federal Reserve Bank of New York

November 2012

Abstract

In the recent financial crisis and the current sovereign debt crisis, there have been large questions surrounding both the health of banks and regulators' ability to provide capital to bail out banks. In a model where the regulator has private information on both, we demonstrate that the regulator may manipulate information flows. First, we show that a regulator with ample resources may forbear on bad banks rather than bail them out to signal toughness and minimize subsequent risk taking by banks. Influencing bank perceptions resolves the moral hazard problem without the need to commit to a "no bailout" policy. Second, a regulator may influence depositor perceptions as well; it can hide bad banks, shielding them from immediate runs. This will come at the cost of having to wastefully inject capital into healthy banks in bad times. Third, a regulator with few resources may provide excessive capital to bad banks to increase confidence and prevent future runs. Lastly, we show that stress tests will be more informative when regulators have capital for bailouts or when market beliefs are negative.

*We thank Mitchell Berlin, Elena Carletti, Brian Coulter, Frederic Malherbe, Juan Francisco Martinez, Alan Morrison and the audiences at the Bank of England, EUI, LBS FIT working group, ISCTE, and U. Porto for helpful comments. The views expressed in this paper represent those of the authors and not necessarily those of the Federal Reserve Bank of New York or the Federal Reserve System.

[†]Saïd Business School, University of Oxford. Contact: Joel.Shapiro@sbs.ox.ac.uk
[‡]Federal Reserve Bank of New York. Contact: David.Skeie@ny.frb.org

Keywords: bank regulation, bailouts, reputation, financial crisis, sovereign debt crisis

JEL Codes: G01, G21, G28

"If money isn't loosened up, this sucker could go down,"

- Statement by former President George W. Bush, quoted in the New York Times on September 26, 2008[1]

1 Introduction

In the quote above, former President George W. Bush highlights the uncertainty in the U.S. over whether funds would be released to resolve the banking crisis in 2008. Uncertainty over the health of financial institutions *and* whether the regulator will act to stabilize those institutions were elements of both the subprime crisis and the ongoing European sovereign debt crisis. In this paper, we examine the role of uncertainty about a regulator's ability to inject capital and how regulators manipulate information in response.

We build a theoretical model where a bank regulator has two sources of private information. The first source is the losses that each bank will incur in an adverse macroeconomic scenario. The second is the regulator's own ability to provide capital to struggling banks. We think of this as a cost of funding that incurs deadweight losses. The amount of funding necessary will depend on the size of banks, which may be very large compared to the tax base (for example, the banking crises of Ireland and Iceland had this feature). The political cost of establishing and tapping new bailout funds (as in the example above and in the European case) adds to the loss.[2]

The regulator can determine the capital each bank must receive (if necessary) to prevent default in the adverse scenario, and can decide to inject capital, liquidate the bank, or forbear. In the model, the regulator must resolve two banks in succession. Depositors run on the bank if the bank is expected to be insolvent. Banks that experience a run or end up insolvent impose costs on all agents. The second bank may engage in moral hazard if

[1] "Talks Implode During a Day of Chaos; Fate of Bailout Plan Remains Unresolved" by David M. Herszenhorn, Carl Hulse, and Sheryl Gay Stolberg, *New York Times*, September 26, 2008. As one may notice from the title of this article, the day was rife with drama and uncertainty.

[2] During the eurozone crisis, countries have been resorting to scrambling for diverse means of outside support; at various times, the EU, ECB, IMF, private equity firms, and even China have entered the conversations. Hoshi and Kashyap (2010) detail how crippling it was for the Japanese government to attempt to use taxpayer funds to assist the banking sector.

it senses that a bailout is guaranteed. Both banks and depositors can learn about the regulator's type through her previous actions. This creates room for manipulating perceptions about the regulator's type through information management.

The regulator's actions with the first bank serve two roles: to resolve the bank and to signal its type. This gives rise to several important results. First, the regulator may want to reduce the perception that it has the ability to bail out banks so as to minimize subsequent risk taking by banks. It can do this by forbearing on a bank it knows to be bad. This is a reputational explanation for a regulator acting tough to diminish moral hazard, as opposed to the commonly found assumption in the literature (discussed below) that a regulator may commit to not conduct bailouts. While it is hard to isolate this effect, it is reminiscent of the Lehman Brothers episode and tough talk from top German leadership about Eurozone bank bailouts in 2010 and 2011.

Second, the regulator may take advantage of the uncertainty about its type in good times to influence depositors and stave off runs at bad banks. This will come at a cost; the regulator will have to wastefully inject some capital into good banks when the market has a negative outlook. The motivation for this injection is a signaling one, as the regulator can indicate which banks are good by injecting enough capital to be costly, but not enough for a bailout. This sheds new light on the TARP capital injections, which had been commonly conjectured as serving to hide the types of some banks - we show that capital injections can be used to separate the types of banks.

Third, the regulator may also inject excessive capital into bad banks in order to build a reputation among depositors for easy access to funding that can prevent future runs. This appears to have been the case of Ireland, who guaranteed their banks despite the fact that the banking sector was too large to effectively do so. The guarantees did prevent runs until the Irish government was given a lifeline by the European Union.

Lastly, we find that credible stress tests are more likely to come from well funded regulators and a regulator with insufficient access to funds is more likely to perform a credible stress test when beliefs about the banking system are negative. This gives some insight to the observable differences in quality between the stress tests in the U.S. and those in Europe. The available capital and the ability to use it allowed U.S. regulators to conduct an effective and credible stress test.

In the model we represent regulator types by how costly it is for the regulator to raise funds for capital injections. Government funding is, of course, important; Demirgüç-Kunt and Huizinga (2010) show that a larger fiscal balance (government revenues minus spending) decreases bank CDS spreads. At the same time, there is also great uncertainty about what governments will do even if they have the capital - Acharya, Drechsler, and Schnabl (2011) document significant decreases in bank CDS spreads after the initial wave of bailouts[3] in the U.S. and Europe (from 9/26/2008 to 10/21/2008). Government actions then led to learning about the government's position.[4]

There is a theoretical literature that examines regulator decisionmaking around the closing and bailing out of banks. The closest paper to ours is Morrison and White (2011), who argue that a regulator may choose to forbear when she knows that a bank is in danger of failing, because liquidating the bank may lead to a poor reputation about the ability of the regulator to screen and trigger contagion in the banking system. We also have potential contagion through reputation, but examine the preferences and resources of a regulator rather than its skill for screening. In addition, we explicitly model bailouts and asymmetric information about the regulator's type. Boot and Thakor (1993) also find that bank closure policy may be inefficient due to reputation management by the regulator, but this is due to the regulator being self-interested rather than being worried about contagion. In their model, the regulator has private information about its screening ability and there are no bailouts. Cordella and Levy Yeyati (2003) focus on the moral hazard dimension, where a regulator must balance being tough and committing no bailouts (avoiding moral hazard) with allowing for bailouts (increasing long run bank value through insurance). Keister (2011) also discusses the difference between committing to no bailouts and allowing the regulator discretion to use bailouts in the context of bank runs and liquidity. He allows for the government to use its remaining funds on public goods, which drive his novel results. Mailath and Mester (1994) discuss credible bank closure policies in a model with full information and without bailouts. Acharya and Yorulmazer (2007, 2008) examine the idea of "too many to fail" and show that because a regulator will use bail outs when many banks are failing, banks will herd in their risk taking.

[3] In their Figure 6. They state, "The bailouts typically conisted of asset purchase programs, debt guarantees, and equity injections or some combination thereof" (p.22).

[4] In that same Figure 6, Acharya, Drechsler, and Schnabl (2011) point out that sovereign CDS spreads increase right after the initial bailouts. As they correctly state, there is a transfer of risk to sovereigns. Nevertheless, our paper suggests there may have been a learning effect as well about the sovereign's resources.

A previous literature has examined the need for regulators to disclose information about the health of banks. DeYoung et al. (1998) and Berger and Davies (1994) find empirical evidence suggesting that banks disclose good news but look to hide bad news, which is revealed because of bank exams by regulators. However, Prescott (2008) develops a model to argue that too much information disclosure by a bank regulator leads to less information that the regulator can gather on banks. Bouvard, Chaigneau, and de Motta (2012) have results similar to ours on stress tests. The show that transparency is better in bad times and opacity is better in good times. However, they do not consider the policy tools of the regulator to conduct bailouts, liquidate banks or forbear and therefore don't look at stress tests in conjunction with regulator responses. They instead focus on liquidity and diversification choices by financial institutions. Peristiani et al. (2010) show that markets had largely identified the distribution of weaker and stronger banks before the 2009 US stress test was conducted, but the stress test provide new information about the size of capital needs among the weaker banks. Hirtle et al. (2009) highlight that the 2009 US stress test was credible and stabilizing for the banking system because the standard microprudential process of analyzing individual bank loss exposures was combined with a macroprudential focus of the need for broad financial stability and the upfront commitment to provide capital to banks.

We proceed as follows. Section 2 sets up the model. In Section 3, we examine the resolution of the second bank. In Section 4, we proceed to examine reputation dynamics by focusing on the resolution of the first bank. In Section 5, regulators can announce information through stress tests. In Section 6, we conclude. All proofs are in the appendix.

2 The Model

There are three types of risk neutral agents: the regulator, banks, and depositors. We will look at the sequential resolution by the regulator of two banks. In addition, there is an interim period after the resolution of the first bank and before the resolution of the second bank where the second bank may choose to risk shift.

We break the setup of the model into four parts: banks and depositors, the regulator's choice set, the regulator's types, and moral hazard.

2.1 Banks and Depositors

For each bank, there are three stages: a resolution decision by the regulator, a withdrawal decision by the depositors, and the realization of the state of nature with its consequent payoffs.

A bank has one unit of an asset. In the third stage, the aggregate state of the world is revealed to be either high returns where all bank assets pay off $\bar{R}$, or low returns, where the bank's assets pays off $\underline{R}_\theta$, where θ is the type of the bank and $\theta \in \{G, B\}$. From an ex-ante perspective, the high returns state occurs with probability q. All agents have a prior over the type of the banks, α, that represents the probability that a bank is good (G) where $1-\alpha$ represents the probability that the bank is bad (B).

There are a mass one of depositors in each bank, who have each deposited 1 unit. For a solvent bank, the exogenous return promised on deposits is $\tilde{R}$ if they are withdrawn at stage 3.[5] The promised return is 1 if deposits are withdrawn earlier (at stage 2). We assume that the bank liquidates its long term asset at stage 2 if necessary. The liquidated asset provides a return of 1.[6] If the bank is insolvent at any stage, the asset return is equally divided among all withdrawing depositors at that stage. The remaining value of each bank is paid to equityholders at stage 3. We assume the following ordering on returns:

$$\bar{R} \geq \underline{R}_G \geq \tilde{R} \geq 1 > \underline{R}_B. \tag{A1}$$

The good bank can always pay depositors the promised return on deposits, while the bad bank won't be able to in the bad state. The return promised to depositors for keeping their money in the bank is larger than that for withdrawing it.

At stage 2, if depositors of a bank expect not to get a return at least as much as their outside option of 1, there is a run and they withdraw their money from the bank immediately, leaving the bank insolvent. We assume

[5] The return $\tilde{R}$ can be set optimally before the stage game begins. For example, in an ex-ante stage, $\tilde{R}$ can be set large enough so that the expected return to the depositors equals their outside option of 1, as in Acharya and Yorulmazer (2007, 2008). In the ex-ante stage, there is a positive probability of entering into the "crisis" stage game we describe here and a positive probability of entering into a game where there are no shocks that would make banks insolvent.

[6] We could allow this return to be lower than 1. In that case, there would be multiple equilibria where self-fulfilling bank runs occur, but we could get similar results by focusing on the equilibria which have fundamentals-based runs.

that if depositors knew a bank was bad, meaning that in the low returns state it would have a bad shock, they would run at stage 2:

$$q\tilde{R} + (1-q)\underline{R}_B < 1$$

We assume that there is no deposit insurance. From condition A1, if depositors know that the bank is good, then they would not run.

In order to define the beliefs of depositors, it is useful first to define a benchmark. We denote α^* as the probability that a bank is good when depositors are indifferent between a run and keeping their money in the bank. Specifically, α^* is defined by:

$$q\tilde{R} + (1-q)(\alpha^*\tilde{R} + (1-\alpha^*)\underline{R}_B) = 1 \tag{1}$$

If there is no run but the bank cannot fully pay depositors at stage 3, the bank is insolvent. We assume that there is a cost C to society per bank that is insolvent or liquidated by the regulator.[7] The cost may represent the loss of value from future intermediation the bank may perform, the cost to resolve the bank, or the cost of contagion. We further assume a cost $C_{run} > C$ if a bank is made insolvent by a run. This would be more costly because of the need to immediately liquidate the long term asset (for example, it could reduce the value of the asset for other agents holding it). While there are the threat of runs in the model, in equilibrium there are no runs, so this cost is not incurred.

2.2 The Regulator's Choice Set

The regulator costlessly observes the type of a bank. The regulator then may take action based on its findings. It has three possible actions: to inject capital, liquidate the bank, or do nothing (forbearance). Injecting an amount of capital X costs λX, where λ is larger than 1 and represents the deadweight loss of raising government funds.[8] The regulator's objective function is to

[7]The insolvency cost may be different in stage 2 versus stage 3, as in stage 2 it occurs because of liquidation, while in stage 3 it occurs because of a bad shock. To simplify matters, we maintain it is the same in both periods. Mailath and Mester (1994) have a similar cost.

[8]Laffont and Tirole (1993) label this the "shadow cost of public funds". As we discuss in the introduction, this could also incorporate the size of the banking sector relative to the tax base and the political costs of using taxpayer money to help banks.

maximize the sum of the expected surplus of all agents minus the cost of insolvencies and potential capital injections.

In order to streamline our presentation, we will assume that regulators prefer to stop both runs and insolvency at a bank, rather than only stopping a run (and permitting a possible insolvency).[9] This reduces the number of cases to consider and streamlines the presentation. Assumption A2 formalizes this:

$$C > \frac{(\tilde{R}-1)(\lambda-1)}{(1-q)^2(1-\alpha)} \tag{A2}$$

This does not imply we are ruling out insolvency - an insolvency will be the best solution for the regulator if the cost of raising capital for the regulator is too high.

When a bank is good, there will be no runs or capital injections. The expected surplus of the regulator (S_G) for the good bank is equal to:

$$S_G = (q\bar{R} + (1-q)\underline{R}_G)$$

When a bank is bad, it may be subject to runs and the regulator may inject capital. How much capital does a bank need? In order to prevent both a run and an insolvency in stages 2 and 3, it needs to inject $X_I = \tilde{R} - \underline{R}_B$. The surplus to the regulator from preventing an insolvency by injecting X_I is:

$$S(X_I) = q\bar{R} + (1-q)\underline{R}_B - (\lambda-1)(\tilde{R} - \underline{R}_B) \tag{2}$$

The surplus to the regulator if a bank is liquidated is equal to $1-C$, the value of the liquidated asset minus the insolvency cost C.

When there is no information, an outcome where a bad bank has no run, no capital injection, and is not liquidated may arise. This occurs if the regulator can effectively "hide" the type of the bad bank through forbearance, i.e. the regulator (i) does not pursue a course of action to prevent potential default of a bank that it knows may be bad and (ii) a run is not provoked. This gives the regulator a payoff of:

$$S_F = q\bar{R} + (1-q)\underline{R}_B - (1-q)C \tag{3}$$

[9] If the insolvency is prevented, the run will also be prevented, but the reverse does not hold true.

We make the following assumption on the parameters throughout the paper:

$$S_F > 1 - C \tag{A3}$$

This assumption puts us in the important case where the regulator would prefer to hide the type of the bank rather than liquidate it. This is at the heart of the information problem.

We suppose that the regulator knows the type of the bank, but depositors do not. Actions taken by regulator takes are observable, and may provide the market with signals. Doing nothing also sends a signal to the market, with all of its consequent implications. In section 5, we will allow the regulator to directly communicate information about banks through stress tests.

2.3 The Regulator's Type

We define the type i of the regulator in terms of the cost of raising funds λ_i. The regulator has three possible choices: bailouts, liquidations, and forbearance. The different costs of funding will create different rankings of the choices for the regulator. Therefore one interpretation of types is that they are literally funding costs, i.e. some regulators may have easy access to funds, but some may face a deadlocked political system and find that the tap is dry. The access to funds may be from outside sources, such as a super-regulator or another sovereign. A more expansive interpretation is that types represent the preferences of the regulator. Regulators may prefer to avoid bailouts at all costs, or may be quick to turn on the fire hose.

The low cost regulator, who has cost of capital λ_L, can afford to inject capital and strictly prefers to do so, preventing the possibility of a costly future bankruptcy:

$$S_L(X_I) > S_F$$

A regulator with high costs is defined as a regulator whose cost of capital λ_H is large enough so that it can avoid liquidations, but prefers to hide the type of the bank to injecting capital:[10]

[10] One might imagine a third type of regulator whose costs are so high that it prefers to liquidate rather than bail out. In a previous version of the paper, we analyze this type of regulator in more detail. Furthermore, in the moral hazard section, it will become clear that the high cost type may become this third type of regulator if there is risk shifting.

$$1 - C < S_H(X_I) < S_F$$

2.4 Moral Hazard

We suppose that the second bank, if it is bad, can risk shift.[11] This choice is observable but non-verifiable. It can increase expected returns in the good state while reducing expected returns in the bad state. Specifically, it can increase $\bar{R}$ to $\bar{R}'$ while simultaneously reducing $\underline{R}_B$ to $\underline{R}'_B$.[12] For simplicity, we restrict this to be a discrete choice (the bank can choose between $(\bar{R}, \underline{R}_B)$ and $(\bar{R}', \underline{R}'_B)$) and make the shift mean-preserving (set $\underline{R}'_B = \underline{R}_B - \frac{q(\bar{R}' - \bar{R})}{1-q}$). The bank maximizes its risk-neutral payoff. As its downside is limited, the bank has a strong incentive to risk shift. We will demonstrate that the bank's choice will depend on the regulators's actions.

Expected payoffs to the bank and the regulator when there is risk shifting to $(\bar{R}', \underline{R}'_B)$ are summarized in the following table:

Regulator action	Payoff to Bad Bank	Surplus for type i Regulator
Bailout	$V_b = q(\bar{R}' - \tilde{R})$	$S_i(X'_I)$
Liquidation	$V_l = 0$	$1 - C$
Forbearance	$V_F = q(\bar{R}' - \tilde{R})$	S_F

where $X'_I = X_I + (\underline{R}_B - \underline{R}'_B)$.

It is worth noting here that by our definition of surplus, the fact that the bank risk shifts alone does not affect our measure of surplus (it is just a transfer of wealth), except for the fact that it induces the regulator to wastefully pump in more money in a bailout.

The decision of the bank to shift risk will impact the expectations of depositors. This implies that there is a different cutoff for when depositors

[11] The equityholders of a good bank could potentially use this tactic as well, but as long as it does not impact regulatory decision making (driving the low return for a good bank $\underline{R}_G$ below $\tilde{R}$), it will not affect our results. Of course, this may be less likely to occur at a good bank because of better governance and monitoring in place.

[12] Similar results would arise if the bad bank's equity holders were able to extract cash payouts such as dividends. Dividend payouts from weak financial institutions were rampant in the early part of the recent financial crisis (see Acharya, Gurjal, Kulkarni, and Shin, 2011).

decide to run. We denote the cutoff when equityholders risk shift as α'^{*}, which is defined by:

$$q\tilde{R} + (1-q)(\alpha'^{*}\tilde{R} + (1-\alpha'^{*})\underline{R}'_B) = 1$$

It is obvious from the above that $\alpha'^{*} > \alpha^{*}$.

A key condition for moral hazard to have bite is:

$$S_H(X_I') < 1 - C \tag{A4}$$

This condition says that diversion of cash flows will cause the high cost regulator to prefer liquidation to a bailout. This means that the bad bank lowers the payoffs and changes the behavior of the high cost regulator. This creates a risk for bad bank as it would obviously prefer to be bailed out rather than liquidated.

For the low cost regulator, we assume that:

$$S_L(X_I') > S_F \tag{A5}$$

This implies that the low cost regulator still prefers bail out a bad bank when there is risk-shifting. In an earlier version of the paper, we consider the case where $S_F > S_L(X_I') > 1 - C$. The qualitative results were similar. We do not include this case for brevity.

2.5 Summary of Timing

There are two banks which the regulator sequentially resolves. As described in the previous subsection, the second bank (if it is bad) may risk shift before its stage game begins. The timing of the basic game for resolving each bank is as follows:

- Stage 1: After observing the type of the bank, the regulator may make a capital injection, liquidate the bank, or forbear.
- Stage 2: Depositors decide whether to run or not. If they run, they take their cash and invest it in their outside option for a return of 1.
- Stage 3: The aggregate state of the world is publicly revealed, assets pay off, and insolvencies costs may occur. Depositors collect their returns, either from the bank (if there was no run) or from the outside option (if

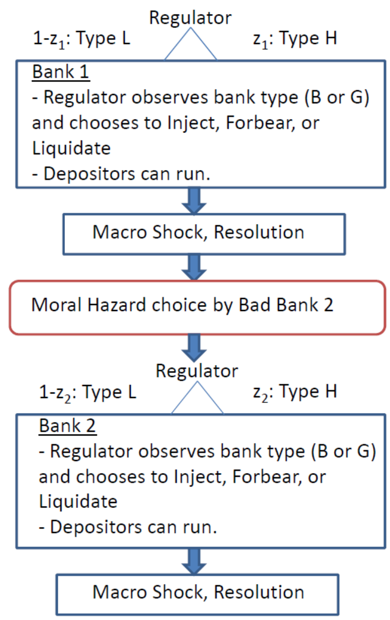

Figure 1: Timing

there was a run). Equityholders collect the remaining returns, if there are any.

We illustrate the timing in Figure 1.

Given that there are two banks to resolve sequentially, there is a discount factor δ for the payoffs from the resolution of the second bank. For simplicity, we do not allow for discounting within the resolution of a given bank.

We assume that the regulator does not know the type of the second bank when resolving the first bank and that this type is independent of the first bank's type. We further assume that the ex-ante probability of having a good bank is α for both banks and that the types of the regulator and the types of the bank are independent.[13]

[13] In reality, it may be the case that the type of the regulator and the type of the bank are correlated. The regulator's function outside of times of crisis is supervising and screening banks. If its ability to supervise and screen is related to its funding (or both are explained by institutional framework), then it can be the case that the quality of the banking system is related to its funding.

We use the concept of Perfect Bayesian Equilibrium and focus on pure strategies.

3 The Second Bank

We begin with the second bank, using backward induction.

Depositors are uncertain about the type of the regulator. They have an ex-ante belief that with probability $1 - z_2$, the regulator prefers to bail out troubled institutions (is type L). With probability z_2, the regulator is believed to have high costs for capital (type H). These beliefs will depend on the inference based on what that regulator did with the first bank and on the ex-ante beliefs z_1.

The decision of the regulator on how to resolve the second bank depends on whether the bad bank decided to risk-shift. If the bad bank did so, the regulator would have to inject extra capital if it conducted a bailout. From A4 and A5, this would change the preferences of both types of regulator.

We apply the intuitive criterion of Cho and Kreps (1987) in solving for the equilibrium strategy of the regulator:

Proposition 1 *For the second bank:*

1. *If there was no risk-shifting:*
   (a) *If* $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha^*$*: There is an equilibrium where the high cost regulators of both types of bank pool with the low cost regulator of the good bank and forbear. The low cost regulator of the bad bank injects* X_I*.*
   (b) *If* $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha^*$*: There is a unique equilibrium where both types of regulator of the good bank provide a capital injection of* X^{**} *(where* $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$*), and both types of regulator of the bad bank inject* X_I*.*

2. *If there was risk-shifting:*
   (a) *If* $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha'^*$*: There is an equilibrium where the high cost regulators of both types of bank pool with the low cost regulator of the good bank and forbear. The low cost regulator of the bad bank injects* X_I'*.*

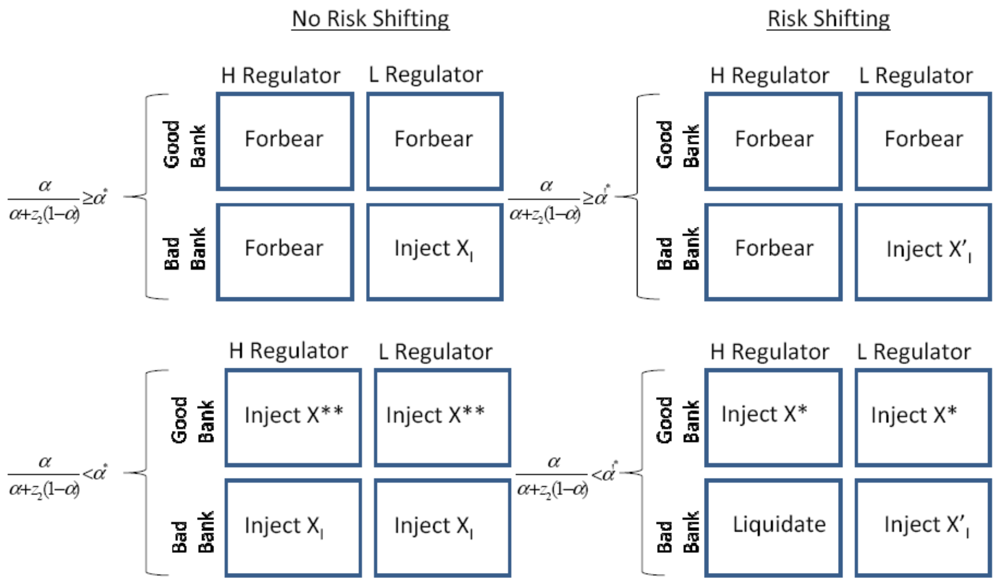

Figure 2: Results of Proposition 1

(b) *If* $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha'^*$*: There is a unique equilibrium where both types of regulator of the good bank provide a capital injection of* X^* *(where* $S_F - (\lambda_H - 1)X^* = 1 - C$*), the high cost regulator of the bad bank liquidates the bank and the low cost regulator of the bad bank injects* X_I'*.*

We depict the results in Figure 2.

In parts 1a and 2a, the equilibrium is not unique. We select the only equilibrium that holds for all off-the-equilibrium path beliefs. At the same time, this equilibrium is the lowest cost one.

Clearly, the beliefs of depositors matter for the regulators. If depositors have favorable beliefs about the health of banks, i.e. the probability of a good bank is above α^*(or above α'^* if there is risk shifting), they will not run. In that case, the high cost regulator will forbear on a bad bank and hide its weaknesses. If depositors have negative beliefs, the regulators will take action in advance to resolve the banks rather than face a run. In this scenario, both regulators will inject capital into the good bank as a show of faith (thereby separating from the bad bank).

There are clear inefficiencies when the type of the bank is unknown and the regulator can only signal through taking steps to resolve a bank. The inefficiencies are well defined. Both regulators have to inject X^* or X^{**} of capital into a good bank when beliefs are unfavorable. This injection is just costly enough to keep the high cost regulator with a bad bank indifferent between mimicking and bailing out the bad bank. This wasteful injection clearly results from asymmetric information about the bank's health.

Intriguingly, there are efficiencies from the type of the bank being unknown. This is because in the situation where beliefs are favorable the high cost regulator can forbear on the bad bank rather than liquidate or inject capital. Forbearing creates a larger surplus for the high cost regulators (S_F).

The presence of inefficiencies and efficiencies depend on the perception of depositors about whether the regulator is high cost (z_2), as this determines their belief about whether the bank is good ($\frac{\alpha}{\alpha+z_2(1-\alpha)}$). This will influence the behavior of the regulator when resolving the first bank, who may benefit by altering this perception.

The capital injections into the good bank by both regulators when market beliefs are pessimistic have a flavor of the initial TARP injections, where several banks received capital injections when they did not need it (e.g. J.P. Morgan)[14]. The commonly held view is that by injecting all of the largest financial institutions with capital, the U.S. regulators were trying to hide which banks were bad. Our model provides a different perspective on the capital injections. Here, the injections are not to hide bad banks, but to let the market know which banks will not fail.

Hiding the bad bank through forbearing is a strategic choice of the regulator in our model. There is little direct evidence on regulators hiding information about banks, but recent events provide indirect evidence. The recent Libor scandal revealed that Paul Tucker, deputy governor of the Bank of England, made a statement to Barclays' CEO that was interpreted as a suggestion that the bank lower its Libor submissions.[15]. Hoshi and Kashyap (2010) discuss several accounting rule changes that the government of Japan

[14] The injections were for the nine largest U.S. banks and summed to $250 billion (see "Drama Behind a $250 Billion Banking Deal", by Mark Landler and Eric Dash, *New York Times*, October 14, 2008).

[15] Barclays then CEO wrote notes at the time on his conversation with Tucker, who reportedly said, "It did not always need to be the case that [Barclays] appeared as high as [Barclays has] recently." This quote and a report on what happened appear in the Financial Times ("Diamond lets loose over Libor", by Brooke Masters, George Parker, and Kate Burgess, Financial Times, July 3, 2012).

used to improve the appearance of financial institutions during their crisis.

4 Information Management and the First Bank

In this section, we analyze the actions the regulators take for the first bank. In its final form, this will bear a similarity to models of reputation building, a la Kreps and Wilson (1982) and Milgrom and Roberts (1982). Here however, we have two sources of asymmetric information, the type of the regulator and the health of the bank. We also do not have a "behavioral" type player, as both regulator types will play rationally given their preferences. We will see that reputation can be used for very different purposes.

We begin by considering the bad bank's risk-shifting choice. Suppose that $\frac{\alpha}{\alpha+z_2(1-\alpha)} > \alpha'^*$. In this case, there are two possible choices for the bad bank, to do nothing or risk shift. We define the payoffs below:

$$\begin{aligned} \text{no change} &: q(\bar{R} - \tilde{R}) \\ \text{risk shift} &: q(\bar{R}' - \tilde{R}) \end{aligned} \tag{4}$$

Choosing risk shifting clearly dominates.

Now suppose that $\frac{\alpha}{\alpha+z_2(1-\alpha)} \leq \alpha'^*$. The payoffs are:[16]

$$\begin{aligned} \text{no change} &: q(\bar{R} - \tilde{R}) \\ \text{risk shift} &: (1 - z_2)q(\bar{R}' - \tilde{R}) \end{aligned} \tag{5}$$

The bad bank would prefer no change if $z_2 > \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$ and risk shift otherwise. This makes sense as the bad bank is only willing to risk shift if there is a high probability that the regulator is low cost.

We now can calculate expected continuation benefits to each type of regulator conditional on the perception z_2. Define the following indicator functions:

$$\begin{aligned} p_1(z_2) &= I_{\frac{\alpha}{\alpha+z_2(1-\alpha)} > \alpha'^*} \\ p_2(z_2) &= I_{\alpha^* < \frac{\alpha}{\alpha+z_2(1-\alpha)} \leq \alpha'^*} \\ \eta(z_2) &= I_{z_2 > \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}} \end{aligned}$$

[16] Note that the payoffs are the same for the interval $[\alpha^*, \alpha'^*]$ and $[0, \alpha^*]$.

The function p_1 is equal to 1 (and zero otherwise) when the belief that the bank is good, given pooling of the regulator types (both regulators with the good bank, the high cost regulator with the bad bank) and the belief that the regulator is high cost (z_2), is above α'^*, the threshold for a run when there is risk shifting. The function p_2 is equal to 1 (and zero otherwise) when the belief is between α'^* and α^*. The function η is equal to 1 (and zero otherwise) when the belief that the regulator is high cost, z_2, is larger than $\frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, which indicates that the second bad bank will not engage in risk-shifting.

Using this framework, we first demonstrate that the equilibrium behavior by the regulator for the resolution of the second bank when there is no risk-shifting is equilibrium behavior for the resolution of the first bank.

Proposition 2 *The equilibrium regulator behavior for the second bank (when there is no risk-shifting) is an equilibrium for the first bank, i.e.*

1. *If $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha^*$: The high cost regulators of both types of bank pool with the low cost regulator of the good bank and forbear. The low cost regulator of the bad bank injects X_I.*

2. *If $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha^*$: Both types of regulator of the good bank provide a capital injection of X^{**} (where $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$), and both types of regulator of the bad bank inject X_I.*

Sufficient conditions for this equilibrium to exist are that $\hat{z}_2 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ (where $\hat{z}_2 \equiv \frac{z_1(\alpha+(1-\alpha)q)}{z_1(\alpha+(1-\alpha)q)+(1-z_1)\alpha}$), $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, and $p_1(\tilde{z}) = 0$, where $\tilde{z}$ is the off-the-equilibrium path belief that the regulator is high cost, and α is not too large.

This equilibrium is not the only equilibrium. The behavior of the regulator at the first bank has two effects beyond the resolution of the first bank. First, it sends a signal to depositors about the type of the regulator and its consequent ability to resolve the second bank. Second, it sends a signal to the second bank, which will subsequently decide whether to risk shift or not. Therefore the regulator has strong incentives to manipulate information about its type.

4.1 Information Management by the Low Cost Regulator

Moral hazard is a key risk discussed by policymakers when bailouts are considered.[17] The argument is that saving a bank today may imply that banks in the future will likely be saved, which will encourage those banks to take excess risks. This suggests that a commitment device which prevents the regulator from discretionary bailouts may be needed to prevent moral hazard. However, we will now show that such a commitment device is not needed, as the regulator can prevent moral hazard by creating uncertainty about its ability to conduct bailouts.

In the model, if the cost of a bailout is too high, bad banks would actually be liquidated. Therefore a bank will not risk shift if that act makes it less likely to be saved. In this case, a low cost regulator may want to pretend to be a high cost regulator in order to reduce moral hazard at bad banks. Even though a low cost regulator may not be able to avoid the commitment problem associated with bailouts, it may use information management to mitigate it.

The low cost regulator can reduce the perception that it is low cost by forbearing on the first bad bank. We demonstrate this in the following proposition:

Proposition 3 *At the first bank, there is an equilibrium where the regulators of the bad bank pool at choosing to forbear when beliefs are favorable. Specifically,*

1. *If $\alpha \geq \alpha^*$: Both types of regulator of the good bank forbear and both types of regulator of the bad bank forbear.*

2. *If $\alpha < \alpha^*$: Both types of regulator of the good bank provide a capital injection of X^{**}(where $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$), and both types of regulator of the bad bank inject X_I.*

Sufficient conditions for this equilibrium to exist are that δ, α, and C are large, $\tilde{z} < \frac{\bar{R}' - \bar{R}}{\bar{R}' - \bar{\tilde{R}}}$, $p_1(\tilde{z}) = 0$, and either $z_1 \geq \frac{\bar{R}' - \bar{R}}{\bar{R}' - \bar{\tilde{R}}}$ or $p_1(z_1) = 1$, where $\tilde{z}$ is the off-the-equilibrium path belief that the regulator is high cost.

[17] Keister (2011) summarizes and adds to the discussion.

By forbearing, the low cost regulator of the bad bank is able to increase the perception that it is a high cost regulator, as it fully separated itself by bailing out the bad bank in Proposition 2. This is useful to the low cost regulator if it prevents risk-shifting. In order to do so it must satisfy two conditions. First, it must be that the belief of the second bank that the regulator is high cost is sufficiently large. Second, if the low cost regulator deviates to a bailout, the second bank must interpret this as likely having come from the low cost regulator, and then decide to risk shift. This implies a restriction on the off-the-equilibrium path beliefs, which we detail in the appendix.

The result states that with the possibility of risk shifting, we are more likely to see bad financial institutions left to the markets at the beginning of a crisis, rather than receiving injections. While there were many things going on at the time, certainly future risktaking factored into the U.S. government's decision to not save Lehman Brothers. Furthermore, the Lehman decision was seen as critical for the market to learn about the government's preferences. Similarly, there has been less intervention in Europe than one might expect. The popular media has designated the position of the European leadership as trying to "muddle" through. While Germany seems like it could act, it repeatedly mentioned legal restraints on itself, the Euro-zone, and the ECB. Could this tough talk be a play to reduce future moral hazard?

4.2 Information Management by the High Cost Regulator

Consider the incentives of the high cost regulator. The benefit for the high cost regulator of revealing its type is that it may stop risk shifting by the second bank. The cost for the high cost regulator of revealing its type is that depositors become more wary since the high cost regulator prefers to hide bad banks, making it more likely that a run will occur. In the following proposition, we show that there is an equilibrium where the high cost regulator mimics the low cost regulator to reduce the perception that it is high cost. It does this by bailing out the first bad bank when beliefs about the bank's type are favorable:

Proposition 4 *At the first bank, there is an equilibrium where the regulators of the bad bank pool and inject X_I into the bad bank when beliefs are favorable. Specifically,*

1. *If $\alpha \geq \alpha^*$: Both types of regulator of the good bank forbear and both types of regulator of the bad bank inject X_I.*

2. *If $\alpha < \alpha^*$: Both types of regulator of the good bank provide a capital injection of X^{**}(where $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$), and both types of regulator of the bad bank inject X_I.*

Sufficient conditions for this equilibrium to exist are that $S_H(X_I)$ is close to S_F, $\tilde{z} < \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, $p_1(\tilde{z}) = 0$, and either $z_1 \geq \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$ or $p_1(z_1) = 1$, where $\tilde{z}$ is the off-the-equilibrium path belief that the regulator is high cost.

By bailing out the first bank when it is bad and saving it from insolvency, the high cost regulator pools with the low cost regulator. This reduces the threat of runs at the second bank, saving both on the cost of bailing out a second bad bank and the cost of injecting capital into a good bank. Essentially, it is building a reputation for its willingness to bailout banks so that depositors will trust that it will do the right thing with the second bank. Interestingly, this result could be stronger if there was contagion. The current cost of having to bailout a bad bank rather than forbear remains fixed, but the future benefit of the bailout increases when there is contagion. The increase is due to the fact that the regulator's decision conditions on the regulator being faced with a first bank that is bad. With contagion, it is more likely to have a second bank that is bad, which increases the expected benefit of preventing future runs. This is, of course, assuming that contagion does not impact the risk-shifting decision of the second bank.

This behavior is reminiscent of the actions taken by Ireland in their banking crisis. They guaranteed their banks despite the fact that the banking sector was too large to effectively do so. The guarantees did prevent runs until the Irish government was given a bailout package by the European Union.

Another recent example is from October 2011, when it seemed like most European countries (including Germany) wanted to recapitalize their banks. This was likely because they either had the capital to inject into their banks or perhaps wanted to build their reputation for action. However, France protested against a coordinated action and recapitalizing in general.[18] They

[18] The Economist ("Banks face new European stress tests", October 5, 2011) writes that, "The French government signalled it was uncomfortable with the accelerating talk of recapitalisation, insisting its banks did not need help...any state recapitalisation could threaten France's triple A soverign debt rating".

may not only have had larger costs of injecting capital into banks (there was some discussion of France losing its AAA rating), but they especially did not want to establish this as a precedent going forward because of their banks' exposure to Italy and Spain. In this sense, it seems like France couldn't afford to build it's reputation.

5 Stress Tests

During the financial crisis of 2007-09, the United States and the European Union conducted stress tests designed to measure potential bank losses. The results of stress tests in the U.S. were believed to be credible, while those in Europe were not. One explanation is that under TARP, the U.S. had funds available for banks if they were short of capital. This allowed U.S. regulators to provide stress test results that would not trigger bank runs because of concerns over future insolvency. In contrast, Europe was seen as lacking the fiscal unity for regulators to be able to provide capital to banks that would be revealed to have large shortfalls.

In this section we offer an explanation that is supportive of the above view, but is more nuanced. We add an initial stage where the regulator may commit to doing stress tests for both banks. In the initial stage, we will assume the regulator does not know the types of the banks. A stress test, when performed, has a tiny cost and will perfectly reveal the type of the bank to the public. We will interpret this perfect revelation as an effective stress test and the lack of a stress test as either simply that or an ineffective stress test.[19]

When the high cost regulator and the low cost regulator make different decisions (i.e. one chooses to do a stress test and the other does not), this is a separating equilibrium and both depositors and the banks learn the type of the regulator. When they make the same decision, nothing is learned about the type from this pooling. For simplicity, we will assume that $z_1 > \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, which implies that the second bank will not risk shift if it's belief that the regulator is high cost is equal to or larger than the initial prior. We define two conditions which will pin down equilibrium choices:

[19] While stress tests by their nature are inherently noisy, there is not much to be gained in this model by having a stress test that is not on the extreme ends of full or no revelation.

$$(1+\delta)(1-p_1(1)-p_2(1))(\lambda_L-1)X^{**} > \delta(1-\alpha)(S_L(X_I)-S_L(X_I')) \quad \text{(C1)}$$

$$\alpha(\lambda_H-1)X^{**} > \frac{p_1(1)+p_2(1)}{1-p_1(1)-p_2(1)}(1-\alpha)(S_F-S_H(X_I)) \quad \text{(C2)}$$

We will describe the conditions in the context of the results. We focus on pure strategy equilibria.

Proposition 5 *When C1 holds and C2 does not hold, only the low cost regulator performs a stress test.*

The low cost regulator has a lot to gain by doing a stress test. As the stress test reveals the quality of the bank, the low cost regulator can the choose its preferred action: bail out a bad bank and forbear on a good bank. It is able to avoid the cost of asymmetric information, which is having to inject capital into the good bank when depositors beliefs are negative. There is one potential cost of doing the stress test; if the low cost regulator does the stress test, but the high cost regulator does not, the low cost regulator will reveal its type perfectly, triggering moral hazard by the second bank. The above proposition looks at the case where the expected moral hazard cost is smaller than the expected savings on capital injections, which is where C1 holds.

Given that the low cost regulator will perform a stress test, the high cost regulator faces a tradeoff. By doing a stress test, it credibly reveals the type of the good bank, and thus saves having to inject capital into the good bank when depositors' beliefs about the bank's health is negative. However, the stress test also reveals the type of the bad bank, which the high cost regulator prefers to hide when depositors have positive beliefs. This forces the high cost regulator to deal with the problem, and bail out the bad bank rather than forbearing on it. This tradeoff is evident from condition C2, which compares the expected cost of injections into good banks with the expected benefits of hiding the bad banks.

There is also an equilibrium where both types perform stress tests.

Proposition 6 *When C2 holds, both types of regulator will perform a stress test.*

The high cost regulator will now conduct a stress test because the cost of capital injections into good banks is too large. Here the low cost regulator has no incentive to deviate whatsoever, because it is pooling with the high cost regulator and thus avoiding risk shifting. The equilibria in Propositions 5 and 6 hold under the parameter spaces specified. We demonstrate in the Appendix that any other possible equilibrium has a set of off-the-equilibrium-path beliefs for which it will not hold, irrespective of the parameters.

There is another insight from these propositions worth emphasizing. Consider condition C2. As the benefit of the stress test for the high cost regulator is only felt when beliefs are negative and the cost of the stress test is only felt when beliefs are positive, this condition depends on the beliefs about the banks' health. In good times, the high cost regulator prefers not to perform stress tests, while in bad times, it needs to do them to save the good banks. The European stress tests were less informative than the U.S. due to a lack of ready capital. Interestingly, however, the European stress tests have been improving[20], which may be in part due to a deteriorating situation.

Lastly, notice that here the high cost regulator reveals information no matter what it does. If it does the stress test, it reveals the type of the banks. If it does not do the stress test, the market infers that the type of the regulator is high cost.

6 Conclusion

Bank runs are often tied to uncertainty about the health of the bank in question and the regulator's response to perceived weakness. We model the uncertainty about the regulator to be about its ability to bail out banks. This is particularly relevant in the current sovereign debt crisis, where there has been uncertainty both about where the money for bailouts would be coming from and whether political costs could be overcome to actually allow money to be used for bailouts. We demonstrate that regulators can take advantage of this uncertainty. The benefits are clear: by hiding their weakness they can

[20] For example, "the Irish central bank asked asset-management group Blackrock to come up with the worst numbers it could realistically posit, hired BCG to make sure Blackrock was doing its work properly - then added another 28% for good measure to come up with its total estimated capital shortfall."(from "EU Banking Waits on a Knife-Edge", by Geoffrey T. Smith, Wall Street Journal Online, April 7, 2011). Spain has recently discussed emulating the Irish approach.

prevent runs. Further reputation management could prevent future runs or moral hazard.

It would be interesting to extend the model to allow for a richer set of instruments available to the regulator such as forcing banks to raise outside equity or merge. Elaborating on the political economy of the regulator's decision process and allowing for correlation between regulator funding and bank quality would also be worth pursuing.

References

[1] Acharya, V., Drechsler, I., and Schnabl, P. (2011), 'Pyrrhic Victory? Bank Bailouts and Sovereign Credit Risk', New York University, Stern School of Business, working paper.

[2] Acharya, V., Gujral, I, Kulkarni, N. and Shin, H-S (2011), 'Dividends and bank capital in the financial crisis of 2007–2009', Journal of Applied Corporate Finance, forthcoming.

[3] Acharya, V., Yorulmazer, T., (2007). "Too many to fail – an analysis of time-inconsistency in bank closure policies," Journal of Financial Intermediation 16, 1–31.

[4] Acharya, V., Yorulmazer, T., (2008). "Information Contagion and Bank Herding", Journal of Money, Credit and Banking, 40(1), 215-31.

[5] Berger, Allen N. and Sally M. Davies (1994). "The Information Content of Bank Examinations," Finance and Economics Discussion Series No. 94-20. Washington: Board of Governors of the Federal Reserve System.

[6] Boot, A.W.A., Thakor, A.V. (1993). "Self-interested bank regulation," American Economic Review 83, 206–211.

[7] Bouvard, M., Chaigneau, P., and A. de Motta (2012). "Transparency in the financial system: rollover risk and crises", working paper, McGill University.

[8] Cho, I.-K., and D. M. Kreps. (1987). "Signaling Games and Stable Equilibria," Quarterly Journal of Economics, 102, 179-221.

[9] Cordella, T., Yeyati, E.L. (2003). "Bank bailouts: Moral hazard vs. value effect," Journal of Financial Intermediation 12 (4), 300–330.

[10] Demirgüç-Kunt, A., and Huizinga, H. (2010). "Are banks too big to fail or too big to save? International evidence from equity prices and CDS spreads," mimeo, World Bank.

[11] DeYoung, Robert, Mark J. Flannery, W.W. Lang and Sorin M. Sorescu (1998). "The Informational Advantage of Specialized Monitors, The Case of Bank Examiners," Federal Reserve Bank of Chicago Working Paper Series, No. WP 98-4.

[12] Hirtle Beverly, Til Schuermann, and Kevin Stiroh (2009). "Macroprudential Supervision of Financial Institutions: Lessons from the SCAP," Federal Reserve Bank of New York Staff Report 409.

[13] Hoshi, T., and Kashyap, A. (2010) "Will the U.S. bank recapitalization succeed? Eight lessons from Japan", Journal of Financial Economics, 97, 398-417.

[14] Keister, Todd. (2011). "Bailouts and Financial Fragility," mimeo, Federal Reserve Bank of New York.

[15] Kreps, D. M., and R. Wilson, (1982) "Reputation and Imperfect Information," Journal of Economic Theory, 27, 253-279.

[16] Laffont, J-J. and Tirole, J. (1993) *Theory of Incentives in Procurement and Regulation*, MIT Press.

[17] Mailath, G., Mester, L., (1994). "A positive analysis of bank closure," Journal of Financial Intermediation 3, 272–299.

[18] Mailath, G.; Okuno-Fujiwara, M. and A. Postlewaite, (1993) "Belief-Based Refinements in Signalling Games," J. Econ.Theory 60:2, 241–76.

[19] Milgrom, P., and J. Roberts, (1982). "Predation, Reputation, and Entry Deterrence," Journal of Economic Theory, 27, 280-312.

[20] Morrison, Alan D. and Lucy White (2011). "Reputational Contagion and Optimal Regulatory Forbearance," ECB Working Paper Series Number 1196.

[21] Peristiani, Stavros, Donald P. Morgan and Vanessa Savino (2010). "The Information Value of the Stress Test and Bank Opacity," Federal Reserve Bank of New York Staff Report 460.

[22] Prescott, Edward Simpson (2008). "Should Bank Supervisors Disclose Information About Their Banks?" Federal Reserve Bank of Richmond Economic Quarterly 94(1), pp. 1–16.

7 Appendix

7.1 Proof of Proposition 1

Proof. Part 1: No risk-shifting.

Given that there are no reputation considerations (this is the final period), the low cost regulator has a dominant strategy to inject X_I in the bad bank. We will therefore consider the actions of the high cost regulator of both types of bank and the low cost regulator of the good bank. Furthermore, using the Cho-Kreps Intuitive Criterion, any off-the-equilibrium path beliefs must place zero weight on the low cost regulator with the bad bank.

A. The parameter space where $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha^*$: There is a semi-pooling equilibrium where the high cost regulator forbears for both bank types and the low cost regulator forbears for the good bank. None of these types would deviate for any beliefs off-the-equilibrium path (to liquidate or capital injection of size $X < X_I$) or on the equilibrium path (to a capital injection of X_I).

For all other potential equilibria, we consider off-the-equilibrium path beliefs where the probability that a bank is good is above α^*. This is consistent with the restriction on off-the-equilibrium path beliefs above. Consider other semi-pooling equilibria where the 3 types of regulator inject an amount X of capital or liquidate the bank. Each type of regulator would deviate to a zero capital injection. Any possible equilibrium where there is more separation (two types pool or there is no pooling) has a similar profitable deviation for the regulator to a zero capital injection. If there was a possible equilibrium with the high cost regulator of the bad bank as the only regulator type giving a zero capital injection, this would provoke a run, meaning that the regulator would deviate to the lowest cost action that another regulator type was taking.[21]

The semi-pooling equilibrium we found is therefore unique when beliefs off-the-equilibrium path are that the bank is good. It also exists when beliefs off-the-equilibrium path are that the bank is bad. It also satisfies the undefeated criterion of Mailath, Okuno-Fujiwara, and Postlewaite (1993).

B. The parameter space where $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha^*$: Define X^{**} such that $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$. Given that $S_F > S_H(X_I)$, the injection X^{**} is smaller than X_I, i.e. it is not large enough to prevent insolvency. As the high cost regulator with the bad bank would never deviate to $X \geq X^{**}$ for any off-the-equilibrium-path beliefs, the intuitive criterion allows us to set the beliefs such that an X greater than or equal to X^{**} comes from the high cost or low cost regulator with the good bank.

The proposed equilibrium is that both regulators of the good bank inject X^{**} and both regulators of the bad bank inject X_I. Given the definition of X^{**}, the high cost regulator with the bad bank prefers to inject X_I rather than choose X^{**} or liquidate. The regulators of the good bank strictly prefer to inject X^{**} than to inject X_I or liquidate. As long as off-the-equilibrium path beliefs are such that the probability that a deviation of an injection $X < X^{**}$ (including $X = 0$) comes from a regulator with a good bank are below α^*, this is an equilibrium.

There is no other equilibrium where both the high cost and low cost regulator with the good bank pool (and the high cost regulator with the bad bank separates), as the types with the good bank must inject $X \geq X^{**}$ in capital to keep the high cost regulator with the bad bank from deviating. From the beliefs established by the intuitive criterion, these regulators would deviate to inject X^{**}.

Consider a potential equilibrium where the three types pool. If they pool at a capital injection less than X_I, there will be a run, and they would have been better off injecting X_I. However, if they pool at X_I, the regulators with the good bank would deviate to X^{**}.

There are also no equilibria where only the high cost regulator with both the good and bad bank pool (as they would want to emulate the low cost regulator with the good bank) or where the low cost regulator with the good bank and the high cost regulator with the bad bank pool (as they would

[21] Therefore there is no equilibrium for any off-the-equilibrium path beliefs where the high cost regulator with the bad bank does not pool.

want to emulate the high cost regulator with the good bank). There are also no pure separating equilibria, since the regulator types with the good bank would have an incentive to mimic whoever is taking the lowest cost action.

Lastly, consider a possible equilibrium where all four regulator types pool and inject X_I into the bank. The regulators with the good bank would deviate to X^{**}.

Therefore the proposed equilibrium is unique.

Part 2: Risk-shifting.

The preferences of the high cost regulator change when there is risk-shifting, as it now prefers to liquidate rather than conduct a bailout. We assumed in A5 that the low cost regulator's preferences don't change, as it still prefers to conduct a bailout than forbear. Therefore the proof and actions for the equilibrium when $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha'^{*}$ are the same as in Part 1A (except that in Part 1A, the parameter space was $\frac{\alpha}{\alpha+z_2(1-\alpha)} \geq \alpha^{*}$). Similarly, the proof for the parameters where $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha'^{*}$ is analogous to the proof in Part 1B where $\frac{\alpha}{\alpha+z_2(1-\alpha)} < \alpha^{*}$. The only difference is the change in the high cost regulator's preferences. Therefore we define X^{*} such that $S_F - (\lambda_H - 1)X^{*} = 1 - C$. ■

7.2 Proof of part 1 of Proposition 2

Proof. We will examine possible deviations for the four types of regulators when $\frac{\alpha}{\alpha+z_1(1-\alpha)} > \alpha^{*}$.[22] In section 8.3, we examine possible deviations for the four types of regulators when $\frac{\alpha}{\alpha+z_1(1-\alpha)} \leq \alpha^{*}$.[23] We examine each type of regulator in succession.

High Cost Regulator with the bad bank: If $\frac{\alpha}{\alpha+z_1(1-\alpha)} > \alpha^{*}$ and the bad bank had no injection and was not liquidated (the static strategy of forbearing), the bank would go insolvent with probability $1 - q$. If it goes insolvent, the depositors realize that the regulator has high costs with probability $z_2 = 1$. Otherwise, $z_2 = \frac{z_1(\alpha+(1-\alpha)q)}{z_1(\alpha+(1-\alpha)q)+(1-z_1)\alpha}$, which is greater than z_1. We will define $\hat{z}_2 \equiv \frac{z_1(\alpha+(1-\alpha)q)}{z_1(\alpha+(1-\alpha)q)+(1-z_1)\alpha}$. Therefore its payoff from using

[22] We will show in subsequent propositions that other equilibria may exist.

[23] We analyze this in a separate subsection as this proof will be relevant to Propositions 3 and 4.

the static strategy in period 1 is:

$$
\begin{aligned}
&S_F + \delta\{qp_1(\hat{z}_2)(\alpha S_G + (1-\alpha)S_F) \\
&+(1-q)(p_1(1)+p_2(1))(\alpha S_G + (1-\alpha)S_F) \\
&+qp_2(\hat{z}_2)[\eta(\hat{z}_2)(\alpha S_G + (1-\alpha)S_F) \\
&+(1-\eta(\hat{z}_2))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))] \\
&+q(1-p_1(\hat{z}_2)-p_2(\hat{z}_2))(\eta(\hat{z}_2)(\alpha(S_G - (\lambda_H - 1)X^{**}) + (1-\alpha)S_H(X_I)) \\
&+(1-\eta(\hat{z}_2))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C)) \\
&+(1-q)(1-p_1(1)-p_2(1))(\alpha(S_G - (\lambda_L - 1)X^{**}) + (1-\alpha)S_H(X_I))\}
\end{aligned}
\tag{6}
$$

Consider a deviation to injecting X_I (and mimicking the low cost regulator with the bad bank). In this case, $z_2 = 0$. Therefore its payoff from deviating in period 1 is:

$$
S_H(X_I) + \delta(\alpha S_G + (1-\alpha)S_F) \tag{7}
$$

Is the deviation profitable? If $\hat{z}_2 > \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, then we write the payoff from not deviating as the difference between equations 6 and 7 :

$$
\begin{aligned}
&S_F - S_H(X_I) + \delta\{[q(p_1(\hat{z}_2)+p_2(\hat{z}_2)) + (1-q)(p_1(1)+p_2(1))](\alpha S_G + (1-\alpha)S_F) \\
&+[q(1-p_1(\hat{z}_2)-p_2(\hat{z}_2)) + (1-q)(1-p_1(1)-p_2(1))](\alpha(S_G - (\lambda_H - 1)X^{**}) + (1-\alpha)S_H(X_I \\
&-(\alpha S_G + (1-\alpha)S_F)\} \\
= \;& (S_F - S_H(X_I))(1 - \delta[q(1-p_1(\hat{z}_2)-p_2(\hat{z}_2)) + (1-q)(1-p_1(1)-p_2(1))])
\end{aligned}
$$

Using the fact that $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$. This expression is positive.

If $\hat{z}_2 < \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, then we write the payoff from deviating as the difference between equations 6 and 7:

$$
\begin{aligned}
&S_F - S_H(X_I) + \delta\{[qp_1(\hat{z}_2) + (1-q)(p_1(1)+p_2(1))](\alpha S_G + (1-\alpha)S_F) \\
&+q(1-p_1(\hat{z}_2))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C)) \\
&+(1-q)(1-p_1(1)-p_2(1))(\alpha(S_G - (\lambda_L - 1)X^{**}) + (1-\alpha)S_H(X_I)) \\
&-(\alpha S_G + (1-\alpha)S_F)\} \\
= \;& S_F - S_H(X_I) + \delta\{(1-q)(1-p_1(1)-p_2(1))(S_H(X_I) - S_F) \\
&+q(1-p_1(\hat{z}_2))((1-C) - S_F)\}
\end{aligned}
\tag{8}
$$

Using the fact that $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$ and $S_F - (\lambda_H - 1)X^* = 1 - C$.

This expression may be positive or negative. For example, it would be positive if $S_H(X_I) = 1 - C$, but it would be negative if $S_H(X_I) = S_F$. Therefore there can be a beneficial deviation.

Another possible deviation would be $S_{dev} < S_G$, with off-the-equilibrium path beliefs $\tilde{z}$. This give a payoff:

$$\begin{aligned}
&S_{dev} + \delta\{qp_1(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\
&+(1-q)(p_1(1)+p_2(1))(\alpha S_G + (1-\alpha)S_F) \\
&+qp_2(\tilde{z})[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\
&+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))] \\
&+q(1-p_1(\tilde{z})-p_2(\tilde{z}))(\eta(\tilde{z})(\alpha(S_G - (\lambda_H - 1)X^{**}) + (1-\alpha)S_H(X_I)) \\
&+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C)) \\
&+(1-q)(1-p_1(1)-p_2(1))(\alpha(S_G - (\lambda_L - 1)X^{**}) + (1-\alpha)S_H(X_I))\}
\end{aligned} \tag{9}$$

It is obvious that if $\tilde{z} = \hat{z}_2$, there would be no profitable deviation. We now ask whether there could be a profitable deviation if $\hat{z}_2 < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$. For example, if we set $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, we get the following payoff:

$$S_F - S_{dev} + \delta q(p_1(\hat{z}_2) - p_1(\tilde{z}))(S_F - (1-C)) \tag{10}$$

This could be negative if S_{dev} were close to S_F, $p_1(\hat{z}_2) = 0$, and $p_1(\tilde{z}) = 1$.

If instead $\hat{z}_2 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ and $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, then the the payoff to not deviating would be:

$$\begin{aligned}
&S_F - S_{dev} + \delta q\{(p_1(\hat{z}_2) + p_2(\hat{z}_2) - p_1(\tilde{z}))S_F \\
&+(1-p_1(\hat{z}_2)-p_2(\hat{z}_2))S_H(X_I) \\
&-(1-p_1(\tilde{z}))(1-C)\}
\end{aligned} \tag{11}$$

This could be negative if S_{dev} were close to S_F, $p_1(\hat{z}_2) + p_2(\hat{z}_2) = 0$, and $p_1(\tilde{z}) = 1$. It would be strictly positive if $p_1(\hat{z}_2) + p_2(\hat{z}_2) \geq p_1(\tilde{z})$.

Low cost regulator with bad bank: Consider a deviation by the low cost regulator with a bad bank when $\frac{\alpha}{\alpha + z_1(1-\alpha)} > \alpha^*$.

The low cost regulator with the bad bank has the static strategy of injecting X_I. In this case $z_2 = 0$ and the payoff is:

$$S_L(X_I) + \delta(\alpha S_G + (1-\alpha)S_L(X_I')) \tag{12}$$

Consider a deviation to forbearing (and pooling with the high cost regulator with the bad bank). In this case, if the bad bank does not go insolvent, which occurs with probability q, $z_2 = \hat{z}_2$ as defined in Proposition 4. If the bank goes insolvent, with probability $1 - q$, the depositors believe that the regulator has high costs with probability $z_2 = 1$. Therefore its payoff from deviating in period 1 is:

$$\begin{aligned}
&S_F + \delta\{qp_1(\hat{z}_2)[\eta(\hat{z}_2)(\alpha S_G + (1-\alpha)S_L(X_I)) \\
&+(1-\eta(\hat{z}_2))(\alpha S_G + (1-\alpha)S_L(X_I'))] \\
&+(1-q)(p_1(1)+p_2(1))(\alpha S_G + (1-\alpha)S_L(X_I)) \\
&+qp_2(\hat{z}_2)[\eta(\hat{z}_2)(\alpha S_G + (1-\alpha)S_L(X_I)) \\
&+(1-\eta(\hat{z}_2))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))] \\
&+q(1-p_1(\hat{z}_2)-p_2(\hat{z}_2))(\eta(\hat{z}_2)(\alpha(S_G - (\lambda_L-1)X^{**}) + (1-\alpha)S_L(X_I)) \\
&+(1-\eta(\hat{z}_2))(\alpha(S_G - (\lambda_L-1)X^*) + (1-\alpha)S_L(X_I'))) \\
&+(1-q)(1-p_1(1)-p_2(1))(\alpha(S_G-(\lambda_L-1)X^{**}) + (1-\alpha)S_L(X_I))\}
\end{aligned} \tag{13}$$

Is the deviation profitable? If $\hat{z}_2 \geq \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, then we write the payoff from deviating as the difference between equations 13 and 12:

$$\begin{aligned}
&S_F - S_L(X_I) + \delta\{(\alpha S_G + (1-\alpha)S_L(X_I)) \\
&-[q(1-p_1(\hat{z}_2)-p_2(\hat{z}_2)) + (1-q)(1-p_1(1)-p_2(1))]\alpha(\lambda_L-1)X^{**} \\
&-(\alpha S_G + (1-\alpha)S_L(X_I'))\} \\
= \;& S_F - S_L(X_I) + \delta\{(1-\alpha)[S_L(X_I) - S_L(X_I')] \\
&-[q(1-p_1(\hat{z}_2)-p_2(\hat{z}_2)) + (1-q)(1-p_1(1)-p_2(1))]\alpha(\lambda_L-1)X^{**}\}
\end{aligned} \tag{14}$$

This expression is always negative. The expression in curly brackets can be made as positive as possible by setting $\alpha = 0$ and $\delta = 1$. However, that still yields a negative expression of $S_F - S_L(X_I')$.

If, on the other hand, $\hat{z}_2 < \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, then we write the payoff from deviating as the difference between equations 13 and 12:

$$\begin{aligned}
&S_F - S_L(X_I) + \delta\{(1-q)(1-\alpha)(S_L(X_I) - S_L(X_I')) \\
&-q(1-p_1(\hat{z}_2))\alpha(\lambda_L-1)X^* \\
&-(1-q)(1-p_1(1)-p_2(1))\alpha(\lambda_L-1)X^{**}\}
\end{aligned}$$

Not surprisingly, this is also negative (we would expect this to be negative as there is risk shifting here, eliminating the benefit of deviating).

Now consider a deviation to S_{dev}, with off-the-equilibrium path beliefs set to $\tilde{z}$. If we assume that if there is a default, the beliefs will be equal to $z_2 = 1$, the above results still hold, i.e. there are no profitable deviations.

High cost regulator with a good bank: Consider a deviation by the high cost regulator with a good bank when $\frac{\alpha}{\alpha+z_1(1-\alpha)} > \alpha^*$. The payoff from using the static strategy in period 1 is:

$$\begin{aligned}
&S_G + \delta\{p_1(\hat{z}_2)(\alpha S_G + (1-\alpha)S_F) \\
&+p_2(\hat{z}_2)[\eta(\hat{z}_2)(\alpha S_G + (1-\alpha)S_F) \\
&+(1-\eta(\hat{z}_2))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))] \\
&+(1-p_1(\hat{z}_2) - p_2(\hat{z}_2))(\eta(\hat{z}_2)(\alpha(S_G - (\lambda_H - 1)X^{**}) + (1-\alpha)S_H(X_I)) \\
&+(1-\eta(\hat{z}_2))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))\}
\end{aligned} \tag{15}$$

Consider a deviation to injecting X_I (and mimicking the low cost regulator with the bad bank). In this case, $z_2 = 0$. Therefore its payoff from deviating in period 1 is:

$$S_H(X_I) + \delta(\alpha S_G + (1-\alpha)S_F) \tag{16}$$

Is the deviation profitable? As in the case with the high cost regulator with a bad bank, if $\hat{z}_2 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, there is no profitable deviation. If $\hat{z}_2 < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, then we write the payoff from deviating as the difference between equations 15 and 16:

$$\begin{aligned}
& S_G - S_H(X_I) + \delta\{p_1(\hat{z}_2)(\alpha S_G + (1-\alpha)S_F) \\
& +(1-p_1(\hat{z}_2))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C)) \\
& -(\alpha S_G + (1-\alpha)S_F)\} \\
= \; & S_G - S_H(X_I) + \delta(1-p_1(\hat{z}_2))((1-C) - S_F)
\end{aligned} \tag{17}$$

Using the fact that $S_F - (\lambda_H - 1)X^{**} = S_H(X_I)$ and $S_F - (\lambda_H - 1)X^* = 1 - C$.

This expression may be positive or negative. For example, it would be positive if $S_H(X_I) = 1 - C$, but it would be negative if $S_H(X_I) = S_G$. Therefore there can be a beneficial deviation.

Another possible deviation would be $S_{dev} < S_G$, with off-the-equilibrium path beliefs $\tilde{z}$. This give a payoff:

$$\begin{aligned}
&S_{dev} + \delta\{p_1(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\
&+p_2(\tilde{z})[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\
&+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))] \\
&+(1-p_1(\tilde{z}) - p_2(\tilde{z}))(\eta(\tilde{z})(\alpha(S_G - (\lambda_H - 1)X^{**}) + (1-\alpha)S_H(X_I)) \\
&+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))\}
\end{aligned} \tag{18}$$

It is obvious that if $\tilde{z} = \hat{z}_2$, there would be no profitable deviation. We now ask whether there could be a profitable deviation if $\hat{z}_2 < \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$. In order to make one as likely as possible, we set $\tilde{z} > \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$ and get the following payoff:

$$\begin{aligned} &S_G - S_{dev} + \delta\{(p_1(\hat{z}_2) - p_1(\tilde{z}) - p_2(\tilde{z}))S_F \\ &+(1 - p_1(\hat{z}_2))(1 - C) - (1 - p_1(\tilde{z}) - p_2(\tilde{z}))S_H(X_I)\} \end{aligned} \tag{19}$$

This could be negative if S_{dev} were close to S_G and $p_1(\hat{z}_2) = 0$.

Now consider $\hat{z}_2 > \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$ and $\tilde{z} < \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$. The the payoff to not deviating would be:

$$\begin{aligned} &S_G - S_{dev} + \delta\{(p_1(\hat{z}_2) + p_2(\hat{z}_2) - p_1(\tilde{z}))S_F \\ &+(1 - p_1(\hat{z}_2) - p_2(\hat{z}_2))S_H(X_I) \\ &-(1 - p_1(\tilde{z}))(1 - C)\} \end{aligned} \tag{20}$$

This could be negative if S_{dev} were close to S_G, $p_1(\hat{z}_2) + p_2(\hat{z}_2) = 0$, and $p_1(\tilde{z}) = 1$. It would be strictly positive if $p_1(\hat{z}_2) + p_2(\hat{z}_2) \geq p_1(\tilde{z})$.

Low cost regulator with a good bank: Consider a deviation by the low cost regulator with a good bank when $\frac{\alpha}{\alpha + z_1(1-\alpha)} > \alpha^*$.

The low cost regulator with the good bank has the static strategy of forbearing. In this case $z_2 = \hat{z}_2$ and the payoff is:

$$\begin{aligned} &S_G + \delta\{p_1(\hat{z}_2)[\eta(\hat{z}_2)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1 - \eta(\hat{z}_2))(\alpha S_G + (1-\alpha)S_L(X_I'))] \\ &+p_2(\hat{z}_2)[\eta(\hat{z}_2)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1 - \eta(\hat{z}_2))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))] \\ &+(1 - p_1(\hat{z}_2) - p_2(\hat{z}_2))(\eta(\hat{z}_2)(\alpha(S_G - (\lambda_L - 1)X^{**}) + (1-\alpha)S_L(X_I)) \\ &+(1 - \eta(\hat{z}_2))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))\} \end{aligned} \tag{21}$$

Consider a deviation to $S_{dev} < S_G$. Its payoff from deviating in period 1 is:

$$\begin{aligned} &S_{dev} + \delta\{p_1(\tilde{z})[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1 - \eta(\tilde{z}))(\alpha S_G + (1-\alpha)S_L(X_I'))] \\ &+p_2(\tilde{z})[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1 - \eta(\tilde{z}))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))] \\ &+(1 - p_1(\tilde{z}) - p_2(\tilde{z}))(\eta(\tilde{z})(\alpha(S_G - (\lambda_L - 1)X^{**}) + (1-\alpha)S_L(X_I)) \\ &+(1 - \eta(\tilde{z}))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))\} \end{aligned} \tag{22}$$

Is the deviation profitable? If $\hat{z}_2 \geq \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ and $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, then we write the payoff from deviating as the difference between equations 21 and 22 :

$$\begin{aligned}&S_G - S_{dev} + \delta\{(1-\alpha)(S_L(X_I) - S_L(X_I')) \\ &-(1-p_1(\hat{z}_2) - p_2(\hat{z}_2))\alpha(\lambda_L - 1)X^{**} + (1-p_1(\tilde{z}))\alpha(\lambda_L - 1)X^*)\}\end{aligned} \tag{23}$$

This is positive for sure if α is small. ■

7.3 Proof of Proposition 3

Proof. We define $p_1(z_2)$, $p_2(z_2)$, and $\eta(z_2)$ as before. We check if there are deviations from the proposed equilibrium equilibrium when $\alpha \geq \alpha^*$ in the first period. We don't explicitly check for deviations from the equilibrium when $\alpha < \alpha^*$, since this part of the equilibrium is very similar to the second part of the equilibrium in Proposition 2, and that proof is given in subsection 8.3.

Low cost regulator with bad bank: We examine whether the low cost regulator with a bad bank would prefer to deviate to injecting X_I. We set beliefs off-the-equilibrium path that the regulator is high cost to be the probability $\tilde{z}$. Refinements do not allow us to specify beliefs off-the-equilibrium path here. Notice that as the regulator knows that $\alpha \geq \alpha^*$, this implies that $p_1(z) + p_2(z) = 1$.

Consider the payoff on the equilibrium path of forbearing. This implies $z_2 = z_1$, and if there is a default, it still remains the case that $z_2 = z_1$ as both types of regulator are forbearing on the bad bank. The payoff is:

$$\begin{aligned}&S_F + \delta\{p_1(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1-\eta(z_1))(\alpha S_G + (1-\alpha)S_L(X_I'))] \\ &+p_2(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1-\eta(z_1))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))]\}\end{aligned} \tag{24}$$

Deviating to an injection of X_I yields the payoff of:

$$\begin{aligned}&S_L(X_I) + \delta\{p_1(\tilde{z})[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1-\eta(\tilde{z}))(\alpha S_G + (1-\alpha)S_L(X_I'))] \\ &+p_2(\tilde{z})[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))]\}\end{aligned} \tag{25}$$

The continuation payoffs in equations 24 and 25 are essentially the same except for the fact that the depositors' beliefs are different. As we want to show that an equilibrium exists, we will set (i) $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ so that there is risk-shifting off the equilibrium path, and (ii) $z_1 \geq \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$. The condition for the regulator *not* wanting to deviate is that the following expression should be positive:

$$\begin{aligned} & S_F - S_L(X_I) + \delta\{(\alpha S_G + (1-\alpha)S_L(X_I)) - p_1(\tilde{z})(\alpha S_G + (1-\alpha)S_L(X_I')) \\ & -(1-p_1(\tilde{z}))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))\} \\ = \; & S_F - S_L(X_I) + \delta\{(1-\alpha)(S_L(X_I) - S_L(X_I')) + (1-p_1(\tilde{z}))\alpha(\lambda_L - 1)X^*\} \end{aligned}$$

This can be positive if δ and α are large, $p_1(\tilde{z}) = 0$, and X^* is large (S_F and/or C large).

If on the other hand $z_1 < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ and $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, the condition for not wanting to deviate would be:

$$S_F - S_L(X_I) + \delta(p_1(z_1) - p_1(\tilde{z}))(\lambda_L - 1)X^* \tag{26}$$

This can be positive if δ and α are large, $p_1(z_1) = 1$, $p_1(\tilde{z}) = 0$, and X^* is large (S_F and/or C large).

High cost regulator with the bad bank: Now let us ask if it is possible that the high cost regulator with a bad bank when $\alpha \geq \alpha^*$ in period 1 would not deviate from a situation where it pools at forbearing with the low cost regulator. We again use beliefs off the equilibrium path that $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$. As forbearing is the best first period choice for an H regulator with a bad bank, any deviation gives less utility. We denote the H regulator's best deviation by S_{dev}, where $S_{dev} < S_F$. This gives a payoff of:

$$\begin{aligned} & S_{dev} + \delta\{p_1(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\ & +(1-p_1(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))\} \end{aligned} \tag{27}$$

If $z_1 < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, keeping the equilibrium strategy of forbearing gives a payoff of:

$$\begin{aligned} & S_F + \delta\{p_1(z_1)(\alpha S_G + (1-\alpha)S_F) \\ & +(1-p_1(z_1))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))\} \end{aligned} \tag{28}$$

Since $p_1(z_1) + p_2(z_1) = 1$.

The H regulator with the bad bank will not deviate if the difference between equations 28 and 27 is positive:

$$S_F - S_{dev} + \delta(p_1(z_1) - p_1(\tilde{z}))(S_F - (1 - C))$$

This is positive for sure if $p_1(z_1) \geq p_1(\tilde{z})$.

If instead $z_1 \geq \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, the benefit from not deviating is:

$$S_F - S_{dev} + \delta(1 - p_1(\tilde{z}))(S_F - (1 - C)) \tag{29}$$

This is positive.

High cost regulator with the good bank: Now consider the high cost regulator with a good bank when $\alpha \geq \alpha^*$ in period 1. We show it would not deviate from a situation where it pools at forbearing with the low cost regulator. We again use beliefs off the equilibrium path that $\tilde{z} < \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$. As forbearing is the best first period choice for an H regulator with a bad bank, any deviation gives less utility. The H regulator's best deviation is denoted again by S_{dev}. By definition, $S_{dev} < S_G$. This gives a payoff of:

$$\begin{aligned} &S_{dev} + \delta\{p_1(\tilde{z})(\alpha S_G + (1 - \alpha)S_F) \\ &+(1 - p_1(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1 - \alpha)(1 - C))\} \end{aligned} \tag{30}$$

Since $p_1(\tilde{z}) + p_2(\tilde{z}) = 1$.

First, assume $z_1 < \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$. Thus, keeping the equilibrium strategy of forbearing gives a payoff of:

$$\begin{aligned} &S_G + \delta\{p_1(z_1)(\alpha S_G + (1 - \alpha)S_F) \\ &+(1 - p_1(z_1))(\alpha(S_G - (\lambda_H - 1)X^*) + (1 - \alpha)(1 - C))\} \end{aligned} \tag{31}$$

Since $p_1(z_1) + p_2(z_1) = 1$.

The H regulator with the good bank will not deviate if the difference between equations 31 and 30 is positive:

$$S_G - S_{dev} + \delta(p_1(z_1) - p_1(\tilde{z}))(S_F - (1 - C))$$

This is positive for sure if $p_1(z_1) = 1$ and $p_1(\tilde{z}) = 0$ or $p_1(z_1) = p_1(\tilde{z})$.

If $z_1 \geq \frac{\bar{R}' - \bar{R}}{\bar{R}' - \tilde{R}}$, then the benefit from not deviating is:

$$S_G - S_{dev} + \delta(1 - p_1(\tilde{z}))(S_F - (1 - C)) \tag{32}$$

This is positive.

Low cost regulator with the good bank: The last deviation to check is from the L regulator of the good bank.

We again set beliefs off-the-equilibrium path that the regulator is high cost to be the probability $\tilde{z}$.

Consider the payoff on the equilibrium path of forbearing. In this case $z_2 = z_1$. The payoff is:

$$\begin{aligned} &S_G + \delta\{p_1(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1-\eta(z_1))(\alpha S_G + (1-\alpha)S_F)] \\ &+p_2(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1-\eta(z_1))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))] \end{aligned} \tag{33}$$

Notice that as the regulator knows that $\alpha \geq \alpha^*$, this implies that $p_1(z_1) + p_2(z_1) = 1$.

A deviation to S_{dev}, where $S_{dev} < S_G$ by definition, yields the payoff of:

$$\begin{aligned} &S_{dev} + \delta\{p_1(\tilde{z})[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1-\eta(\tilde{z}))(\alpha S_G + (1-\alpha)S_F)] \\ &+p_2(\tilde{z})[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_L(X_I)) \\ &+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))] \end{aligned} \tag{34}$$

Similarly, $p_1(\tilde{z}) + p_2(\tilde{z}) = 1$.

Set $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ and $z_1 < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$. The condition for the regulator *not* wanting to deviate is that the following expression should be positive:

$$S_G - S_{dev} + \delta(p_1(z_1) - p_1(\tilde{z}))(\alpha(\lambda_L - 1)X^* + (1-\alpha)(S_F - S_L(X_I')))$$

This is positive for sure if $p_1(z_1) = 1$ and $p_1(\tilde{z}) = 0$ or $p_1(z_1) = p_1(\tilde{z})$, and α is large.

Now consider $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ and $z_1 \geq \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$. The payoff from not deviating is:

$$S_G - S_{dev} + \delta\{(1-p_1(\tilde{z}))\left(\alpha(\lambda_L - 1)X^*\right) + (1-\alpha)[S_L(X_I) - p_1(\tilde{z})S_F - (1-p_1(\tilde{z}))\,S_L(X_I')]\} \tag{35}$$

Which is positive. ■

7.4 Proof of Proposition 4

Proof. We check if there are deviations from the proposed equilibrium equilibrium when $\alpha \geq \alpha^*$ in the first period. We don't explicitly check for deviations from the equilibrium when $\alpha < \alpha^*$, since this part of the equilibrium is very similar to the second part of the equilibrium in Proposition 2, and that proof is given in subsection 8.3.

High cost regulator with the bad bank: We begin by seeing whether the high cost regulator with a bad bank would prefer to deviate to forbearing. We set beliefs off-the-equilibrium path that the regulator is high cost to be the probability $\tilde{z}$. Note that refinements do not allow us to specify beliefs off-the-equilibrium path here.

Consider the payoff on the equilibrium path of injecting X_I. In this case $z_2 = z_1$. The payoff is thus:

$$\begin{aligned}&S_H(X_I) + \delta\{p_1(z_1)(\alpha S_G + (1-\alpha)S_F) \\ &+(1-p_1(z_1))[\eta(z_1)(\alpha S_G + (1-\alpha)S_F) \\ &+(1-\eta(z_1))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\}\end{aligned} \tag{36}$$

Notice that as the regulator knows that $\alpha \geq \alpha^*$, this implies that $p_1(z_1) + p_2(z_1) = 1$.

Deviating to forbearing implies $z_2 = z_1$ as both regulator types are forbearing on the good bank in equilibrium. However, with probability q the bad bank fails. This is out-of-equilibrium, so we place beliefs $z_2 = \tilde{z}$. The payoff is:

$$\begin{aligned}&S_F + \delta\{(qp_1(z_1) + (1-q)p_1(\tilde{z}))(\alpha S_G + (1-\alpha)S_F) \\ &+q(1-p_1(z_1))[\eta(z_1)(\alpha S_G + (1-\alpha)S_F) \\ &+(1-\eta(z_1))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))] \\ &+(1-q)(1-p_1(\tilde{z}))[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\ &+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\}\end{aligned} \tag{37}$$

As before, $p_1(z_1) + p_2(z_1) = 1$ and $p_1(\tilde{z}) + p_2(\tilde{z}) = 1$.

The condition for the regulator *not* wanting to deviate is that the following expression should be positive:

$$
\begin{aligned}
&S_H(X_I) - S_F + \delta(1-q)\{(p_1(z_1) - p_1(\tilde{z}))(\alpha S_G + (1-\alpha)S_F) \\
&+(1-p_1(z_1))[\eta(z_1)(\alpha S_G + (1-\alpha)S_F) \\
&+(1-\eta(z_1))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))] \\
&-(1-p_1(\tilde{z}))[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\
&+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\}
\end{aligned} \tag{38}
$$

This can be positive, for example, if we set $z_1 < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $p_1(z_1) = 1$, $p_1(\tilde{z}) = 0$, and $S_H(X_I) = S_F$. These conditions are consistent with there being a profitable deviation from the static equilibrium. Alternatively, it can be positive if $z_1 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$,$p_1(\tilde{z}) = 0$, and $S_H(X_I) = S_F$. In this case it equals:

$$
S_H(X_I) - S_F + \delta(1-q)\{S_F - (1-C)\} \tag{39}
$$

Since the above assumptions imply that $p_1(z_1) = 0$.

We look at another deviation to $S_{dev} > S_H(X_I)$, where we assign beliefs off-the-equilibrium path for no default and for defaults to be $\tilde{z}$. The payoff from deviating would be:

$$
\begin{aligned}
&S_{dev} + \delta\{p_1(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\
&+(1-p_1(\tilde{z}))[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\
&+(1-\eta(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\}
\end{aligned} \tag{40}
$$

Setting $z_1 < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, the condition for the regulator *not* wanting to deviate is that the following expression should be positive:

$$
S_H(X_I) - S_{dev} + \delta\{(p_1(z_1) - p_1(\tilde{z}))(S_F - (1-C)) \tag{41}
$$

This would be positive if $S_H(X_I) = S_F$ and $p_1(z_1) = 1$ and $p_1(\tilde{z}) = 0$. Similarly, if $z_1 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ and $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, the conditions for not wanting to deviate would be if $S_H(X_I) = S_F$ and $p_1(\tilde{z}) = 0$.

Low cost regulator with the bad bank: Now let us ask if it is possible that the low cost regulator with a bad bank when $\alpha \geq \alpha^*$ in period 1 would deviate from a situation where it pools at injecting X_I with the high cost regulator. We will consider the best deviation S_{dev}, which gives rise to off-the-equilibrium path beliefs $\tilde{z}$.[24] We also assume that if there is a default,

[24] For example, we could set $S_{dev} = S_F$ and $\tilde{z} = z_1$.

off-the-equilibrium path beliefs remain $\tilde{z}$. We choose $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$. As injecting X_I is the best first period choice for an L regulator with a bad bank, any deviation gives less utility. This gives a payoff of:

$$S_{dev} + \delta\{p_1(\tilde{z})(\alpha S_G + (1-\alpha)S_L(X_I')) \\ +(1-p_1(\tilde{z}))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))\} \tag{42}$$

Keeping the equilibrium strategy of injecting X_I gives a payoff of:

$$S_L(X_I) + \delta\{p_1(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ +(1-\eta(z_1))(\alpha S_G + (1-\alpha)S_L(X_I'))] \\ +p_2(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ +(1-\eta(z_1))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))]\} \tag{43}$$

The L regulator with the bad bank will not deviate if the difference between equations 43 and 42 is positive. If $z_1 < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$,

$$S_L(X_I) - S_{dev} + \delta(p_1(z_1) - p_1(\tilde{z}))\alpha(\lambda_L - 1)X^* \tag{44}$$

This will be positive if we set $p_1(z_1) = 1$, $p_1(\tilde{z}) = 0$ as before.

If $z_1 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$,

$$S_L(X_I) - S_{dev} + \delta\{(1-\alpha)(S_L(X_I) - S_L(X_I')) + (1-p_1(\tilde{z}))(\alpha(\lambda_L - 1)X^*)\} \tag{45}$$

which is strictly positive.

Low cost regulator with the good bank: Now consider the low cost regulator with a good bank when $\alpha \geq \alpha^*$ in period 1. We show it would not deviate from a situation where it pools at forbearing with the high cost regulator. We again use beliefs off the equilibrium path that $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$. As forbearing is the best first period choice for an L regulator with a good bank, any deviation gives less utility. The L regulator's best deviation is denoted again by S_{dev}. By definition, $S_{dev} < S_G$. The payoff from deviating is given by equation 42.

Keeping the equilibrium strategy of forbearing gives a payoff of:

$$S_G + \delta\{p_1(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ +(1-\eta(z_1))(\alpha S_G + (1-\alpha)S_L(X_I'))] \\ +p_2(z_1)[\eta(z_1)(\alpha S_G + (1-\alpha)S_L(X_I)) \\ +(1-\eta(z_1))(\alpha(S_G - (\lambda_L - 1)X^*) + (1-\alpha)S_L(X_I')))]\} \tag{46}$$

The L regulator with the good bank will not deviate if the difference between equations 46 and 42 is positive. If $z_1 < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, this implies:

$$S_G - S_{dev} + \delta(p_1(z_1) - p_1(\tilde{z}))\alpha(\lambda_L - 1)X^*$$

This will be positive if we set $p_1(z_1) = 1$, $p_1(\tilde{z}) = 0$ as before.

If $z_1 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$,

$$S_G - S_{dev} + \delta\{(1-\alpha)(S_L(X_I) - S_L(X_I')) + (1 - p_1(\tilde{z}))(\alpha(\lambda_L - 1)X^*)\} \quad (47)$$

which is strictly positive.

High cost regulator with the good bank: The last deviation to check is from the H regulator of the good bank when $\alpha \geq \alpha^*$ in period 1.

We again set beliefs off-the-equilibrium path that the regulator is high cost to be the probability $\tilde{z}$.

Consider the payoff on the equilibrium path of forbearing. In this case, $z_2 = z_1$. The payoff is thus modified:

$$\begin{aligned} &S_G + \delta\{p_1(z_1)(\alpha S_G + (1-\alpha)S_F) \\ &+(1 - p_1(z_1))[\eta(z_1)(\alpha S_G + (1-\alpha)S_F) \\ &+(1 - \eta(z_1))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\} \end{aligned} \quad (48)$$

Notice that as the regulator knows that $\alpha \geq \alpha^*$, this implies that $p_1(z_1) + p_2(z_1) = 1$.

A deviation to S_{dev}, where $S_{dev} < S_G$ by definition, yields the payoff of:

$$\begin{aligned} &S_{dev} + \delta\{p_1(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\ &+(1 - p_1(\tilde{z}))[\eta(\tilde{z})(\alpha S_G + (1-\alpha)S_F) \\ &+(1 - \eta(\tilde{z}))(\alpha(S_G - (\lambda_H - 1)X^*) + (1-\alpha)(1-C))]\} \end{aligned} \quad (49)$$

Similarly, $p_1(\tilde{z}) + p_2(\tilde{z}) = 1$.

If $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ and $z_1 < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, the condition for the regulator *not* wanting to deviate is that the following expression should be positive:

$$S_G - S_{dev} + \delta(p_1(z_1) - p_1(\tilde{z}))(S_F - (1-C)) \quad (50)$$

This will again be positive if we set $p_1(z_1) = 1$, $p_1(\tilde{z}) = 0$.

If $\tilde{z} < \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$ and $z_1 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$, the condition is:

$$S_G - S_{dev} + \delta(1 - p_1(\tilde{z}))(S_F - (1-C)) \quad (51)$$

This will again be positive if we set $p_1(\tilde{z}) = 0$. ■

7.5 Proofs of Propositions 5 and 6 (Stress Tests)

We will analyze the full game where each type of regulator can choose to perform a stress test or not. We will only look at pure strategy equilibria.

I. The Low Cost Regulator does a stress test, the High Cost Regulator does not

We begin with the low cost regulator's payoff:

$$\alpha S_G + (1-\alpha)S_L(X_I) + \delta(\alpha S_G + (1-\alpha)S_L(X_I')) \tag{52}$$

Since the low cost regulator separates by doing the stress test, its type is recognized and the second bad bank risk shifts. If the low cost regulator were to deviate to not doing a stress test, it would be perceived to be a high cost regulator. Its payoff would then be:

$$\begin{aligned}&(1+\delta)\{(p_1(1)+p_2(1))[\alpha S_G + (1-\alpha)S_L(X_I)]\\&+(1-p_1(1)-p_2(1))[\alpha(S_G-(\lambda_L-1)X^{**})+(1-\alpha)S_L(X_I)]\}\end{aligned} \tag{53}$$

Therefore the condition for the low cost regulator not wanting to deviate is:

$$(1+\delta)(1-p_1(1)-p_2(1))(\lambda_L-1)X^{**} > \delta(1-\alpha)(S_L(X_I)-S_L(X_I')) \tag{54}$$

The high cost regulator's payoff in this scenario is:

$$\begin{aligned}&(1+\delta)\{(p_1(1)+p_2(1))[\alpha S_G + (1-\alpha)S_F]\\&+(1-p_1(1)-p_2(1))[\alpha(S_G-(\lambda_H-1)X^{**})+(1-\alpha)S_H(X_I)]\}\end{aligned} \tag{55}$$

The high cost regulator does not do the stress test, but its type is recognized. It is able to forbear and hide the bad bank when the beliefs about the banks are favorable, but has to inject capital into the good bank to save it from a run when beliefs are negative.

If the high cost regulator deviated to doing a stress test, its payoff would be:

$$(1+\delta)(\alpha S_G + (1-\alpha)S_H(X_I)) \tag{56}$$

It would therefore deviate if the following condition held:

$$\alpha(\lambda_H - 1)X^{**} > \frac{p_1(1) + p_2(1)}{1 - p_1(1) - p_2(1)}(1-\alpha)(S_F - S_H(X_I)) \tag{57}$$

II. The High Cost Regulator does a stress test, the Low Cost Regulator does not

The high cost regulator's payoff is the same as in equation 56. If it were to deviate and not do a stress test, it would be thought of as a low cost type. It's payoff then would be:

$$(1+\delta)[\alpha S_G + (1-\alpha)S_F] \tag{58}$$

where we assume in the case of a first period bad bank that defaults, the investors do not update the type of the regulator (which is consistent with Perfect Bayesian Equilibrium). In this case, the deviation is profitable.

III. Both types do the stress test

Lemma 1 *When both regulators commit to stress tests there is an equilibrium that has both types of regulator forbearing on good banks and injecting X_I into bad banks for both the first and second banks.*

In this case, the identity of the banks are revealed, but the identity of the regulators are not. It is easy to see this is an equilibrium. Given the type of the bank is known, the regulators each choose their preferred action. There is no risk shifting, as we assumed above that $z_1 > \frac{\bar{R}'-\bar{R}}{\bar{R}'-\tilde{R}}$. There are other equilibria sustained by beliefs off-the-equilibrium path that the regulator is low cost with probability one. This equilibrium, however, is the lowest cost one and exists for any beliefs off-the-equilibrium path, and we will focus on it. The payoff for the low cost regulator in this equilibrium is:

$$(1+\delta)(\alpha S_G + (1-\alpha)S_L(X_I)) \tag{59}$$

Obviously, the low cost regulator would not deviate for any beliefs off of the equilibrium path.

The payoff for the high cost regulator is the same as in equation 56. If the high cost regulator were to deviate and not do a stress test, using the intuitive criterion, it would be recognized as a high cost regulator. In this case, its payoff would be that of equation 55. It wouldn't deviate if the condition from equation 57 held.

IV. Neither type does the stress test

Here, we will use the equilibrium found in Proposition 2, where the first bank equilibrium is the same as the equilibrium in the second bank equilibrium if there is no risk-shifting. This builds many of the main intuitions that are also present in applying stress tests to the other two equilibria (Propositions 3 and 4). The payoff for the high cost regulator in this equilibrium is:

$$\begin{aligned}
&[(p_1(z_1)+p_2(z_1))(1+\delta(p_1(\hat{z})+p_2(\hat{z}))) \\
&+(1-p_1(z_1)-p_2(z_1))\delta(p_1(z_1)+p_2(z_1))](\alpha S_G+(1-\alpha)S_F) \\
&+[(p_1(z_1)+p_2(z_1))\delta(1-p_1(\hat{z})-p_2(\hat{z})) \\
&+(1-p_1(z_1)-p_2(z_1))(1+\delta(1-p_1(z_1)-p_2(z_1))](\alpha(S_G-(\lambda_H-1)X^{**})+(1-\alpha)S_H(X_I))
\end{aligned} \tag{60}$$

where we define $\hat{z}_2 \equiv \frac{z_1(\alpha+(1-\alpha)q)}{z_1(\alpha+(1-\alpha)q)+(1-z_1)\alpha}$ as in Proposition 2.
The payoff for the low cost regulator is:

$$\begin{aligned}
&(1+\delta)[\alpha S_G+(1-\alpha)S_L(X_I)] \\
&-\alpha(\lambda_L-1)X^{**}\{(p_1(z_1)+p_2(z_1))\alpha\delta(1-p_1(\hat{z})-p_2(\hat{z})) \\
&+(1-p_1(z_1)-p_2(z_1))(1+\delta(1-p_1(z_1)-p_2(z_1)))\} \\
&-\delta(1-\alpha)^2(S_L(X_I)-S_L(X_I'))(p_1(z_1)+p_2(z_1))
\end{aligned} \tag{61}$$

where we have re-arranged terms. The last line represents the fact that if there are favorable beliefs and a bad bank in the first period, the low cost regulator bails it out and reveals itself to be low cost. This reveals the low cost regulator's type, which leads to risk-shifting.

Consider off the equilibrium path beliefs where the regulator is believed to be high cost for sure. In this case, the low cost regulator will deviate as it will be able to forbear on a good bank and bail out a bad bank, without risking risk-shifting.