The Use of Derivatives in Financial Engineering: Hedge Fund Applications

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Over the past fifteen years, the term "financial engineering" has come to be widely accepted as describing a way of thinking about and addressing financial issues in all areas of finance including corporate finance, asset management, investment finance, and financial institutions. Depending on their particular niche within the field, the term *financial engineering* means different things to different people. But in general financial engineering can accurately be described as "the development and creative application of financial technology to solve financial problems and exploit financial opportunities."[1] Financial engineering makes use of heavy duty quantitative tools, the uses for which were once thought to be limited to physics and engineering. But also included in the financial engineer's tool kit is the entire spectrum of financial instruments. Perhaps most important among these financial instruments are derivatives.

There are many applications of financial engineering in corporate finance. Many of these applications are actually structured by financial engineers working for banks who "pitch" their "solutions" to corporate managements and boards. Examples would include the use of currency swaps to obtain funding in currencies other than a corporation's domestic currency, perhaps to fund foreign operations, or funding in currencies other than the domestic currency and then swapping into the domestic currency because funding costs are cheaper in the foreign currency (even after allowing for the cost of the swap). Other applications include hedging the risks that corporations are exposed to including interest rate risk, credit risk, commodity price risk, and so forth.

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[1] This was the original definition of financial engineering adopted by the International Association of Financial Engineers (IAFE), the first professional society specifically for financial engineers, when it was launched in 1992.

In other cases, by changing the method by which a corporate objective is achieved, or the products used to achieve it, there can be favorable accounting and/or tax implications.

Not only are many of the applications of derivatives forms of financial engineering, but the very design and analysis of new financial products, including new types of derivatives, are forms of financial engineering. In asset management, financial engineers structure new financial products to better appeal to the risk-reward appetites of investors. For example, there are investors who like to buy short term notes but who want floating rates instead of fixed rates, or who want the performance of the note linked to the performance of some other asset class such as equity (i.e., equity-linked notes), or gold or oil (i.e., commodity-linked notes), or to a specific credit that differs from the issuer's credit (i.e., credit-linked notes). They will even structure products that pay off based on which of several assets or asset classes performs better over a period of time.

Other, relatively simple, asset management applications include the use of asset swaps by fund managers to convert equity portfolios to synthetic fixed income portfolios and vice versa based on a temporary change in a strategic asset allocation plan. For example, a U.S. pension fund manager who held a diversified equity portfolio in early 2000, and who felt (correctly) the market was in a bubble, might have wanted to move from stocks to bonds for a while, say two years. But that would necessitate selling stock, buying bonds, holding the bonds for two years, then selling the bonds and buying stock (to return to the original strategic asset allocation). But, by simply entering into a properly structured two-year asset swap, the fund manager could likely have achieved exactly the same economic result with considerably less transaction cost.

The handiwork of financial engineers, whether they are using that title or not, shows up in all variety of risk management applications, most efforts at funding cost reduction, the development of more efficient trading platforms (such as electronic markets), and on and on. While financial engineers wear many different hats, they share certain characteristics in how they approach problems. For example, one key characteristic of financial engineers is how they look at financial opportunities. Most often, they see a financial opportunity as a bundle of risks. They look at each risk to determine whether or not they want to bear it. Then, they systematically hedge away the risks they do not wish to bear. After factoring in the cost of hedging away the risks, they then ask, "is the reward I expect to earn sufficient to justify the risks I am choosing to bear?"

Successful hedge fund managers are generally competent financial engineers, or employ competent financial engineers, who apply their talents to the world of investment finance.[2] For this reason, it is instructive to see how hedge fund managers might use derivatives in their strategies to extract, what they like to call, "alpha." Two very good examples of this way of thinking about investment opportunities are convertible bond arbitrage and capital structure arbitrage, both are strategies employed by quantitatively sophisticated hedge fund managers who specialize in these areas. In both strategies, derivatives play a key role.

[2] A good example of this new thinking is James Simons, who runs the hugely successful hedge fund management firm known as Renaissance Technologies and who was awarded the 1996 Financial Engineer of Year Award by the International Association of Financial Engineers (IAFE).

Convertible Bond Arbitrage

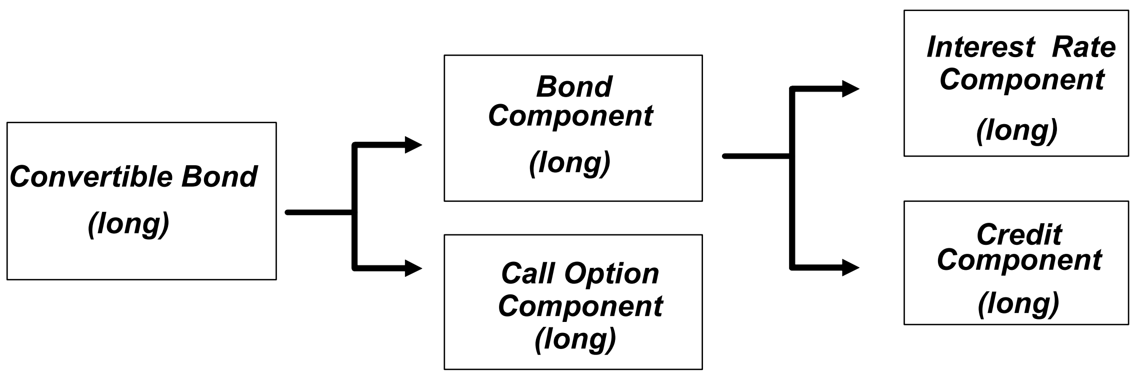
Today, over 90% of new convertible bond issuances (converts) sold in the United States and in Europe are sold directly to hedge funds—usually through private placements. These offerings tend to be large, averaging over a half a billion dollars in the U.S. and even larger in Europe. Because of the size of the deals, a number of hedge funds will each take a portion of an issuance.

It is important to appreciate that most converts are sold by corporations that can accurately be described as weak credits. The reason for this is simple. A weak credit would have to pay a high coupon to sell its debt at par. Further, such corporations generally do not have any collateral to put up, thereby necessitating an even higher coupon. The issuer cannot afford such a high coupon. To get the investor to accept a lower coupon, the issuer offers the investor something else of value, namely a call option on the issuer's common stock.

Consider now a corporation that approaches the corporate finance desk (CFD) of an investment bank. The corporation indicates its desire to issue a convert in the amount of $500 million and a desire to sell it through a private placement. The CFD then refers the inquiry to the private placement desk (PPD). The PPD has a list of hedge funds that are in the market for covvertible bonds for convertible bond arbitrage. So, the PPD calls the hedge funds to ask them if they might be interested in the deal.

If a hedge fund responds that it may be interested, in which case it requires more detail, the PPD sends the hedge fund a non-disclosure agreement (NDA) which the hedge fund must execute and return before the PPD will send the hedge fund the term sheet and other specifics, including the name of the issuer. The NDA states that the hedge fund will not disclose any of the information being provided to any other party and that it will not act on this information if it chooses not to participate in the private placement.

The hedge fund sees the convert as a portfolio of instruments, Specifically, it sees a long position in a convert as a combination of a long position in a corporate bond and a long position in a call option on the same corporation's stock.



The bond component can be viewed as a bundle of risks. Principal among these are "interest rate risk" and "credit risk." There might be other risks if the convert happens to be callable or putable, but we will assume that it is not.

The hedge fund is actually only interested in the call option component of the convert and only then if the option can be acquired "cheap." This brings us to a fundamental principle of option pricing. The value of an option, as long ago shown by Black and Scholes, is a function of five key variables that can be thought of as the value drivers of the option. These are (1) the current price of the underlying stock, (2) the strike price of the option, (3) the time remaining before the option expires (time to expiry), (4) the relevant interest rate for the time to expiry, and (5) the future volatility of the price of the underlying stock. We can see the current price of the stock, we can see the strike price of the option (at least in the types of options that Black and Scholes were modeling), we can see the time to expiry, and we can see the relevant interest rate. But, we cannot see the future volatility of the underlying stock. Thus, when you are trading options, what you are really trading is the one thing you cannot see. You are trading future volatility.

Volatility is measured as the standard deviation of the annual percentage change in the price of the underlying stock. The price of the option (i.e., the option premium) is directly related to the level of the perceived volatility. While the future volatility cannot be seen until after the future has become the past, it can nevertheless be inferred from the price of the option. That is, one can use an option pricing model to "back out the volatility implied by the option's price." Such a volatility is called an "implied volatility." Therefore, the option embedded in the convert is cheap if it can be bought at an artificially low implied volatility.

So how do we determine if the implied volatility is artificially low? This is where the complexity arises. While the offering we have described is for $500 million, we will assume that our hedge fund would only take $100 million. Other hedge funds will take the other $400 million. For purposes of illustration, however, we will do the analysis in terms of $1000 of par value, rather than $100 million.

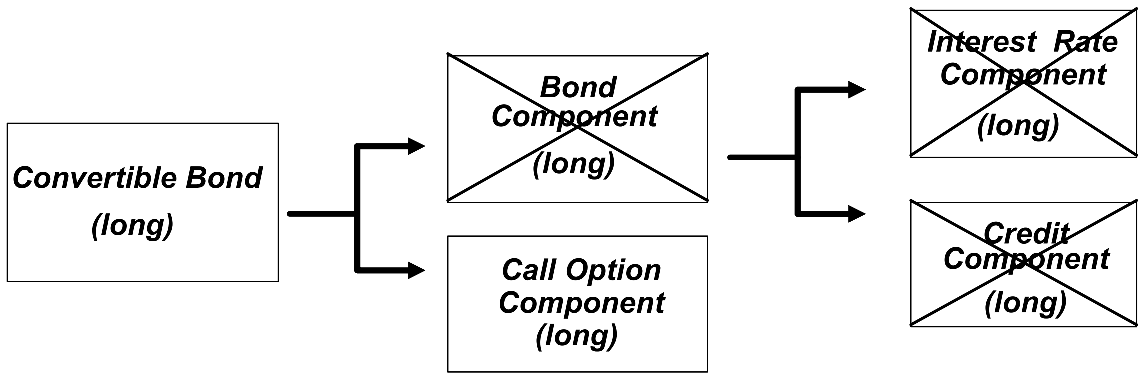
Suppose the term sheet says the following: The convert has a life of 5 years and pays a coupon of 7.25% (for simplicity we will assume that this is paid once a year). The conversion ratio is 20 and does not change over the life of the convert (that is, $1000 of par value is convertible into 20 shares of common stock). The option is only exercisable at the very end of its life (so it is of the European type). The underlying stock does not pay a dividend and isn't expected to pay one over the life of the convert. The stock is currently trading at $40 a share. Because there is no clearinghouse for the embedded option, the relevant interest rate is not quite the risk-free rate.[3] We will use the five-year rate associated with a five-year interest rate swap as the relevant interest rate (this is oversimplifying a bit, but it is a reasonable approximation for our purposes).

The first thing the hedge fund manager would likely ask himself is "what do I think the true future volatility of this company's stock is?" There are several ways that one might be able to come up with an answer to that question. If there are long-dated options on this company's stock trading on one or more of the options exchanges, we could simply back out the implied volatility from those. Or, we might look at the historic realized volatility and use that as an estimate of the stock's future volatility. This requires that we are willing to assume that the past volatility is indicative of future volatility. Or, there might be comparable companies that do have options trading on them and we could use an average of the implied volatilities from those options as a proxy for the future volatility of the subject company's stock. Suppose that one of these methods leads the hedge fund manager to conclude that the subject stock's volatility should be about 35 (i.e., 35%).

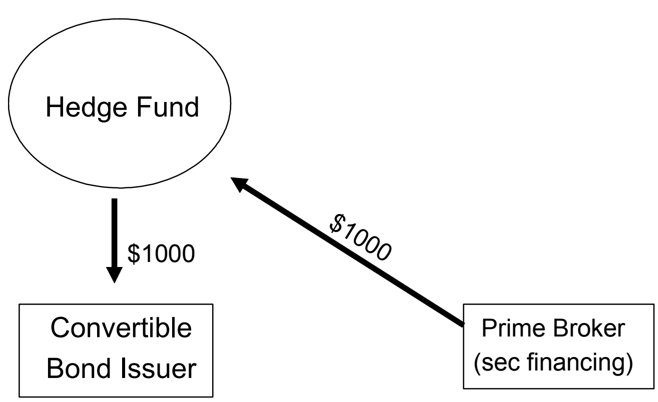
In order to determine if the option is cheap or rich, the hedge fund manager would need to (1) determine the revenue that the convert will generate and the cost of funding

[3] The interest rate plays two distinct roles in the pricing of an option. The risk-free rate (adjusted for the dividend yield) is used to determine the expected future value of the stock. Then an interest rate is used to discount the terminal value of the option to the present value of the option. The latter rate would only be the risk-free rate if there were no counterparty credit risk.

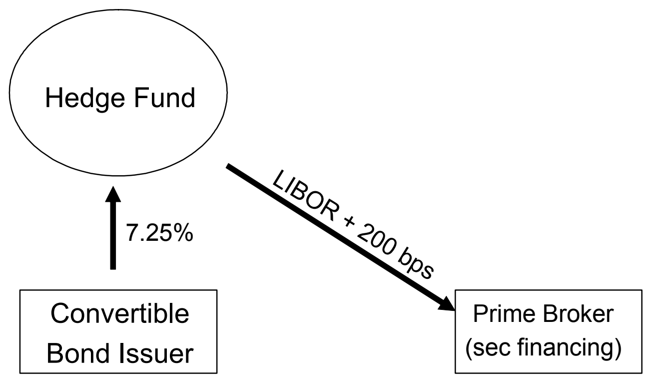
the position, (2) the cost of stripping out everything but the option (meaning he has to get rid of the bond and that means he has to get rid of the interest rate risk and the credit risk), (3) determine the implied price of the option, and (4) from the price of the option derive the implied volatility.



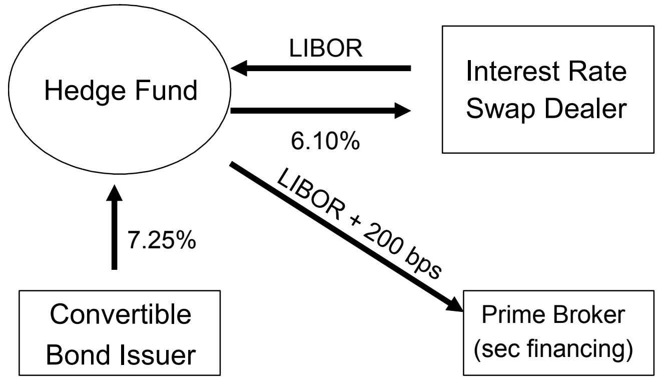
So let's continue with the example we have laid out. The hedge fund manager must first do the requisite analysis, before he can decide if he wants to do the trade. He begins with a series of phone calls. The first call is to the "securities financing facility" of the hedge fund's prime broker. This will be the source of most of the capital for the trade. That is, the hedge fund needs to borrow $1000 to buy the convert. Suppose that the prime broker says that it will lend the money at a rate of LIBOR + 200 basis points (i.e., 2%).



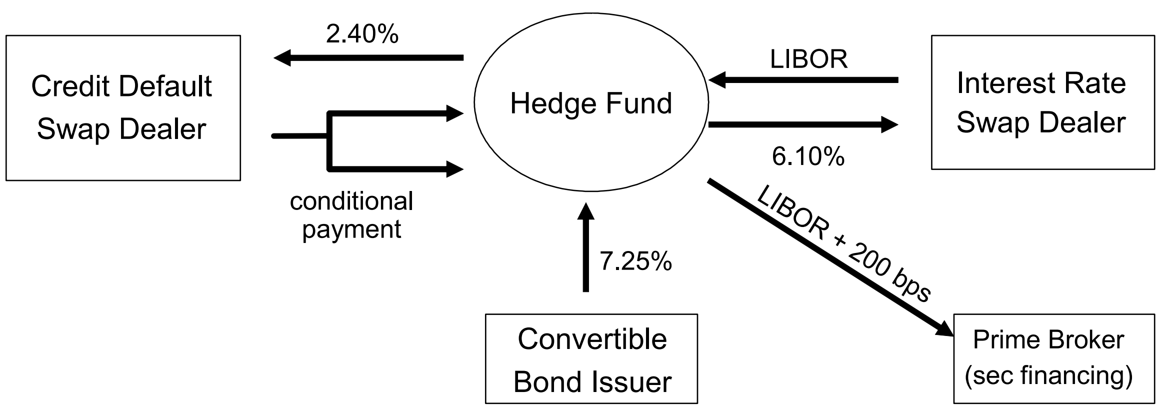
Based on this, the hedge fund manager sees that it has a fixed rate of 7.25% coming in (the coupon on the convert) and a floating rate of LIBOR + 2% going out.



This clearly exposes the hedge fund to considerable interest rate risk. That is, the hedge fund holds a fixed rate asset, but it is funded by a floating rate liability – never a good scenario. To address this problem, the hedge fund manager calls an interest rate swap dealer to price up a five-year plain vanilla interest rate swap in which the hedge fund would be the fixed rate payer and floating rate receiver. Suppose the swap dealer says the hedge fund would have to pay 6.10% against LIBOR flat.



The interest rate swap removes the interest rate risk in the sense that the LIBOR that is being received on the interest rate swap cancels the LIBOR portion of what is being paid to the prime broker. But the hedge fund manager still has the credit risk to deal with. To address this problem, he calls a credit default swap dealer and prices up a plain vanilla five year CDS. Suppose the CDS dealer says that it will sell credit protection for five years at an annual rate of 240 basis points.



Since the interest rate swap eliminates the interest rate risk and the credit default swap eliminates the credit risk, the hedge fund manager has, for all intents and purposes, eliminated the bond component of the convert. He now needs to determine what the option cost. For purposes of this exercise, let's assume that all of the cash flows above are made once a year at year's end. The hedge fund manager will treat monies paid out as costs and monies received as reductions in cost. That is, the cost of the option =

LIBOR + 2% + 6.10% + 2.40% - 7.25% - LIBOR = 3.25%. That is, the option appears to cost 3.25% of the convert's par value. Since the par value is $1000, this is $32.50. However, this is not quite right for two reasons. First, the option covers 20 shares of the stock (i.e., the conversion ratio is 20). Thus we have to divide by 20 in order to get the cost of the option per share covered. This is $1.625. Second, the hedge fund has to pay this each year for five years. So the true cost of the option, when viewed as a premium paid up front, is the sum of the present values of five annual payments of $1.625. Using the interest rate on the swap, 6.10%, as the discount rate, we get a present value of $6.83.

We can now know that the price the hedge fund is paying for the option is $6.83, the life of the option is five years, the option is of the European type and the underlying stock does not pay dividends, the stock is currently priced at $40 a share, and the strike price of the option is $50 per share. The strike price is $50 because the option is only exercisable at the very end of the convert's life at which point the hedge fund has a choice of taking $1000 or taking 20 shares of stock. Thus, the hedge fund "surrenders" $50 for each share of stock it chooses to take. Finally, we will use the interest rate swap rate as a proxy for the five-year interest rate. Since we have all of the value drivers for the option except volatility, we can extract the implied volatility for the stock from a Black/Scholes model.

The result is an implied volatility of 15 (i.e., 15%).[4] Since the hedge fund manager believes that the vol is actually closer to 35 and he can buy it through this rather complex process for 15, he would likely conclude that he can indeed get the option cheap. At this point, it would seem that the hedge fund manager should call everyone

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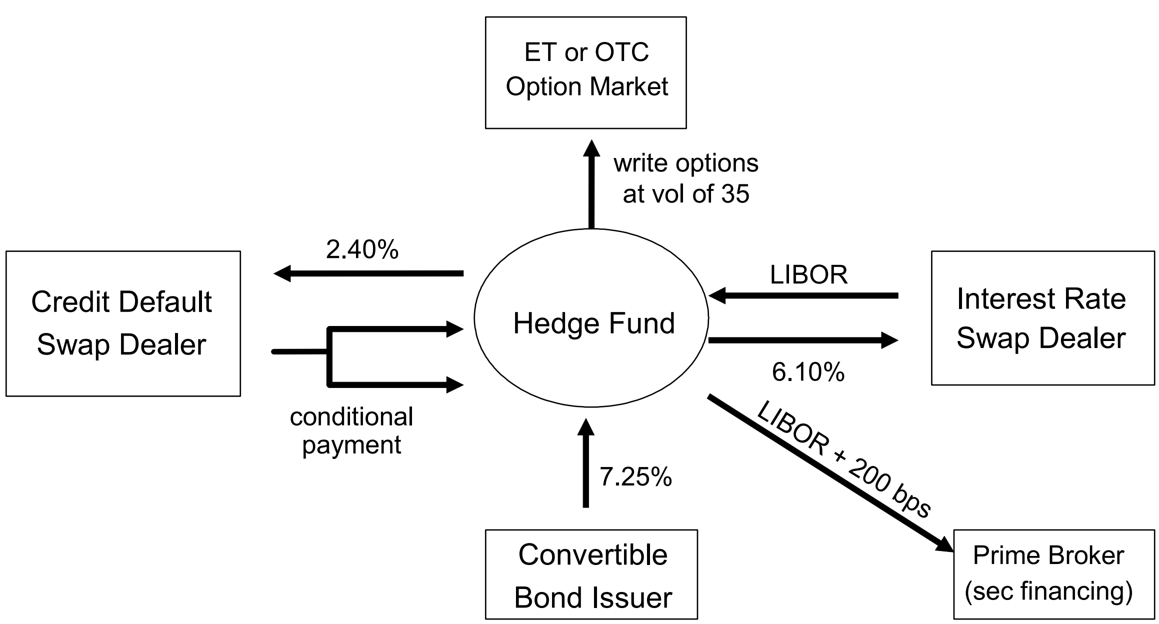
[4] Any good option analytics software package should be capable of extracting this implied volatility. We used an internally developed package for this purpose.

back and say "yes I will take the convert, yes I will take the funding, yes I will do the interest rate swap, and yes I will do the credit default swap—that is he will "pull the trigger on the trade." But we are not quite ready for that. The problem is that he has only managed to buy the option cheap. To be arbitrage, the hedge fund manager must buy the option cheap and simultaneously sell it rich. That is, buy it in one market at a low vol and sell it in another market at a higher vol. So, before he pulls the trigger, he has to determine if, and how, he can sell it rich.

There are two possible ways to sell the option rich. The first, and the easier of the two, is to write call options on this same stock at the higher vol. This, however, is only possible if there are options on the stock trading on an options exchange (i.e., exchange traded or ET) and there is enough depth and liquidity in these options to sell a sufficient number of them at the higher vol. Alternatively, the options can be written in the over-the-counter (OTC) market, but the same liquidity concern applies.



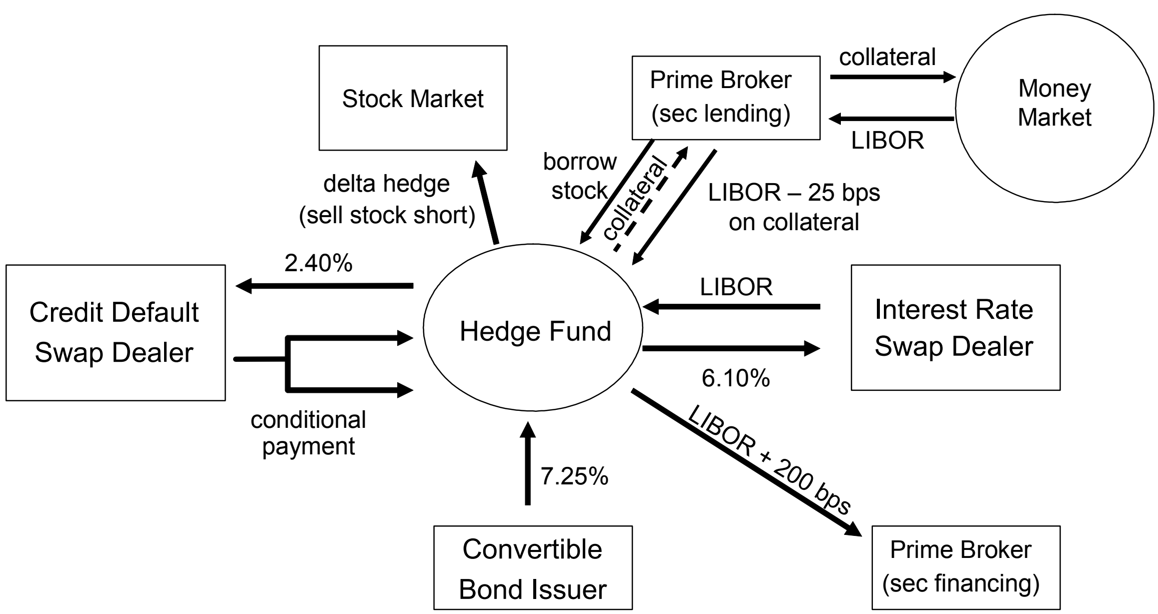
Unfortunately, it is quite likely that that will not be possible either because (1) the options don't trade, or (2) there is insufficient liquidity to sell the number of options the hedge fund manager needs to sell. Recall that the hedge fund manager is not really buying $1000 of par value, but rather $100 million. Other hedge funds, who are using the same strategy will be buying another $400 million. Collectively, the convert issuance is convertible into 10 million shares. That is substantial relative to the size of the options market.

That brings us to the second way to sell the option rich: Create it synthetically. Or, more accurately, replicate its behavior synthetically through a process called "delta hedging." The delta of an option is simply the change in the value of the option that would be caused by a change in the value of the underlying stock. The delta of a call will always be between 0 and +1 (from the perspective of a long position). The more deeply a call is in-the-money, the closer its delta will be to one. The more deeply a call is out-of-the-money, the closer its delta will be to zero.

With the given set of value drivers: stock price is $40, strike price is $50, time to expiry is 5 years, interest rate is 6.10%, and the hedge fund manager believes the true vol to be about 35; we can calculate all of the option's "greeks," of which delta is one. In this case, the option's delta turns out to be +0.69. That is, for each one dollar that the stock price changes, the value of the call will change by $0.69 in the same direction (this ignores the gamma effect).

To delta hedge this option, all the hedge fund manager has to do is recognize that the delta of the stock is always 1.0. That is, the ratio of the change in the price of the stock to the change in the price of the stock is one, because anything divided by itself is one. Thus, the hedge fund manager needs to sell short 0.69 shares of the stock for each share covered by the option. For $1000 of par value, which is convertible into 20 shares, the delta hedge would require that the hedge fund manager go short 13.8 shares (i.e., .69 x 20). For $100 million, the delta hedge would require that the hedge fund manger go short 1,380,000 shares. For the $500 million of the entire offering, the hedge funds collectively will go short 6,900,000 shares of the stock.

So where does the hedge fund manager get the stock to sell short? The answer is from another specialized unit at his prime broker. This unit is called the "securities lending facility." The securities lending facility will arrange to borrow the stock and lend it to the hedge fund. The proceeds from the short sale of the stock (plus a bit more as a cushion) will then be turned over to the prime broker to be held as collateral. The prime broker will invest this money in safe short-term vehicles earning a sum that will approximate LIBOR. The prime broker, however, will keep a little of this for its services and pay the rest out to the hedge fund. Let's suppose the prime broker keeps 25 bps, so the hedge fund gets LIBOR – 0.25%.



The delta hedge above, in which our hedge fund manager borrowed and sold 1,380,000 shares would seem to solve the problem. But we have a few more issues to address. First, the cost of borrowing stock (the 25 bps paid to the prime broker) increases the cost of the option we bought or, equivalently, decreases the price of the synthetic option we sold, and this must be factored in. Second, when this hedge fund and the other hedge funds that participate in this deal collectively sell short 6,900,000 shares of the issuer's stock, the price of the stock is, undoubtedly, going to be impacted. This will drive the price of the stock down with the effect that the delta will change (due to the option's gamma). This gamma effect needs to be taken into consideration when "sizing the delta hedge." Finally, an option's delta is not static. That is, the passage of time will cause the delta of the option to change. Fluctuations in the stock price and fluctuations in the level of interest rates will also cause the option's delta to change. This means that the hedge fund manager must periodically recompute the appropriate delta and adjust the size of the delta hedge. Not to do so would expose the hedge fund to risks it probably does not want to bear. This is why delta hedging is often described as "dynamic delta hedging."

If the convertible bond arbitrage described above is properly executed, and the delta hedge is properly managed, the hedge fund will accrue the value of the difference between the volatility bought and the volatility sold as time passes. Of course, the transaction costs associated with periodically adjusting the delta hedge will eat into this profit to some degree. For this reason, the hedge fund manager will not do the trade unless he perceives the discrepancy between the implied vol in the option embedded in the convert and the true vol to be of sufficient magnitude.

The trade we have been discussing has been around for a number of years now and is reasonably well understood. Some people consider it a type of volatility trading (which it is). Some people consider it a type of capital structure arbitrage (which it is). Some people consider it a type of relative value trade (which it is). And some people just like to think of it as a specific type of trade unto itself.

Capital Structure Arbitrage:

Capital structure arbitrage involves buying one component of a corporation's capital structure (such as its bonds) and simultaneously selling another component of the same corporation's capital structure (such as its stock). By this definition, it is easy to see why some people consider convertible bond arbitrage a form of capital structure arbitrage. As we saw, in convertible bond arbitrage, we bought a corporation's convertible bond and we simultaneously sold the same corporation's stock in an effort to delta hedge the embedded option. Nevertheless, when most people talk about capital structure arbitrage, they are not talking about convertible bond arbitrage.

Capital structure arbitrage rests on two key ideas. The first is conceptually simple, but computationally tedious. We will not delve into the intricacies of the calculations. Rather, we will confine ourselves to the inherent logic.

All other things (i.e., interest rates, coupon payments, recovery rates in the event of default, etc.) being equal, the price of a corporate bond implies the probability that the bond will default over some period of time. This makes intuitive sense. If the price of the bond declines, it must mean that the market has concluded that the probability of default has gone up. Similarly, if the price of the bond rises, it must mean that the market has concluded that the probability of default has gone down.

As it turns out, if we have a number of different bonds from the same issuer, say a one-year bond, a two-year bond, a three-year bond, and so on, we can back out the entire term structure of default probabilities from the bonds' prices. That is, we can determine the probability of a default over the next year, and the probability of a default over the second year, and the probability of default over the third year, and so forth. The process is akin to the extraction of spot zero rates from Treausury bonds via the algorithm known as bootstrapping.

The second key idea is conceptually tricky and also computationally complex unless you are willing to make a number of simplifying assumptions. The idea originated with a paper by Robert Merton published in 1974.[5] In this paper, Merton argued that a credit-risky corporate bond may be viewed as a portfolio consisting of a credit-risk-free bond and a short position in a put option on the company's assets. In 1993, Moody's Investor Services KMV unit (Moody's KMV), showed that Merton's argument implies that a company's common stock can be viewed as a long call on the company's assets. At that moment, the seeds of capital structure arbitrage were planted, but they would still need almost another decade to take root.

Why can we think of a company's common stock as an option on the company's assets? The best way to answer this question is by way of an illustration. Suppose that the accounting values of a company's assets are consistent with the market values of the company's assets. Suppose further that the company has $45 of assets for each share of common stock outstanding and it has $30 of bonds for each share of common stock outstanding. All creditors of the company are holders of the bonds and the bonds take the form of ten-year zero coupon bonds. By the tautology of balance sheets, the book value (but not the market value) of the company's stock must be $15.

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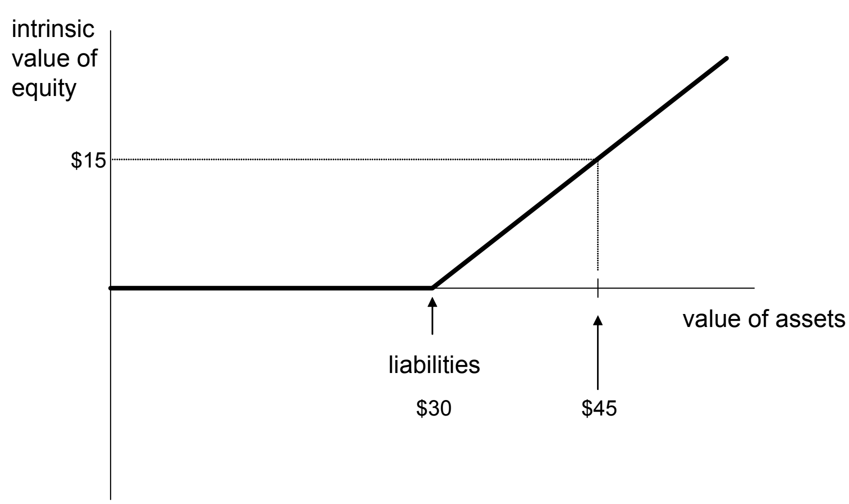
[5] On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance*, 1974, pp. 449-470

Let's assume that the ten-year zero coupon rate of interest is 5.25%, and that the company's stock is trading for $27.38.

Now suppose that the company suffers losses so that the value of the company's assets declines. This will also, of course, result in a corresponding decline in the book value of the company's equity. Now ask yourself this: Could the book value of the company's equity ever become negative? The answer, of course, in an *accounting* sense, is yes. If the value of the company's assets declines below $30 per share outstanding, the accounting book value of the company's stock becomes negative. But now ask a more important question. Can the *economic* book value of the company's stock ever become negative? For the economic book value to become negative, it would mean that the shareholders would willingly pay others to take their stock away from them. This is, of course, nonsense. Shareholders in a corporation have absolute limited liability. The company can simply declare bankruptcy, thereby surrendering the assets of the company to the creditors, i.e., the bond holders. Let's assume that the company would declare bankruptcy the moment the value of its assets declined to the book value of its bonds, or, equivalently, when the book value of its equity reaches zero.



This economic book value is, essentially, the intrinsic value of a call option which is readily apparent from the graphic relationship between the intrinsic value of the equity and the value of the assets. That is, whenever you see a "hockey stick" you know you are looking at an option.

However, we said that the stock is not trading at its intrinsic value of $15. Rather it is trading at its market price of $27.83. Why? The answer is time value. In addition to intrinsic value, options have time value. Time value represents the potential for the option (in this case the stock is the option) to acquire additional intrinsic value prior to the option's expiry.

By thinking of the common stock as an option on the company's assets, it becomes clear that the value of the stock (when viewed as an option) is a function of the usual option value drivers: the value of the underlying assets, the strike price of the option, the time to option expiry, the rate of interest, and the volatility of the underlying assets. We know all these values except the volatility of the assets. But we also know the market price of the option (i.e., the stock), so we can extract the implied volatility of the assets the way we would extract an implied volatility from any option premium.

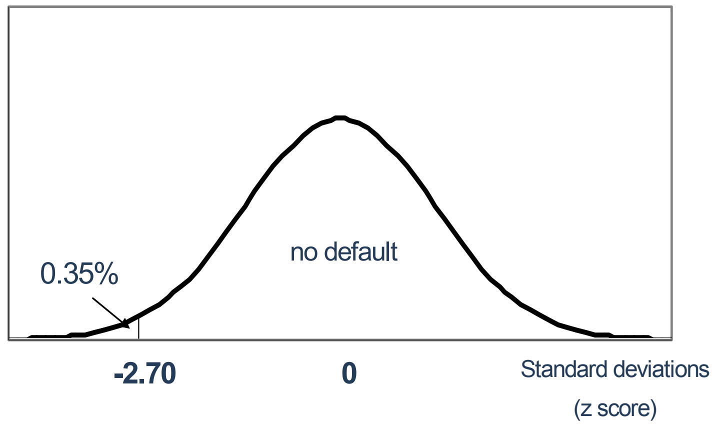
In this case, the market price of the option (i.e., the stock) is $27.83, the underlying asset is at $45, the strike price is $30, the time to expiry is the life of the bonds which is 10 years, and the interest rate is 5.25%. Extracting the implied volatility of the assets, we get a vol of 15 (i.e., an annual vol of 15%).

We now ask ourselves another question. How far would the value of the assets have to decline for the company to declare bankruptcy, which would constitute default on its bonds? Well, we assumed that the company would declare bankruptcy the moment the value of its assets declined to the value of its liabilities, at which point its economic book value would be zero. That would require a decline of $15 in the value of the assets from the current $45 level to $30. We can convert this to a percentage change (continuously compounded) by taking the natural log of the ratio of $30/$45.[6] The result is -40.5%. That is, if the company's asset value declines by 40.5%, the company will default.

Since a "vol" is simply a standard deviation, we can now ask the question "how many standard deviations would the value of the assets have to decline in order to result in default?" This is just -40,5% divided by 15%, which is -2.70 standard deviations.

Assuming the percentage change in the value of the assets is approximately normally distributed, we can now just calculate the cumulative probability from negative infinity to a z score of -2.70. We get the result 0.35%.

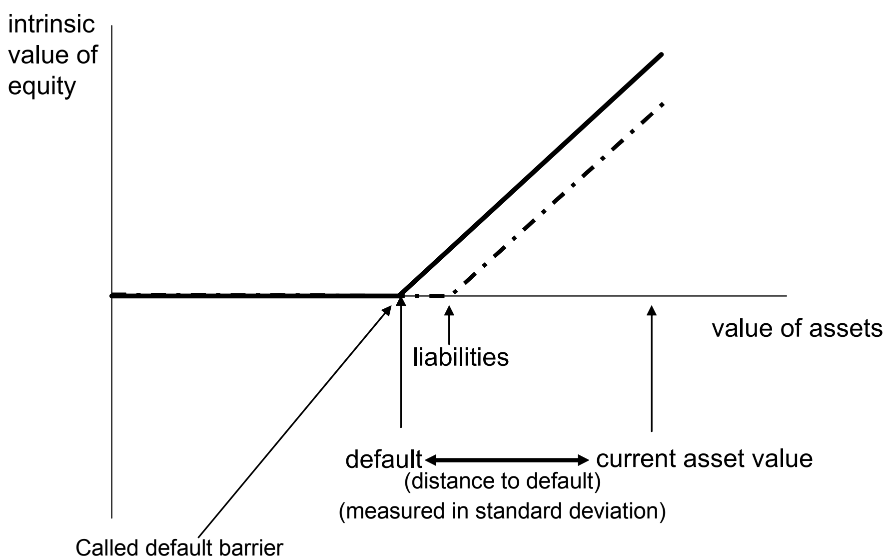
[6] When asset prices are lognormally distributed, the percentage change in their value will have a normal distribution when that percentage change is measured on the assumption of continuous compounding. Additionally, an asset's volatility (the vol) is routinely measured on the assumption of continuous compounding.



We can therefore conclude that there is 0.35% chance that this company will default on its bonds within a one year period. We can extend this to get the probability of default over a two year period, a three year period, and so on to derive the entire term structure of default probabilities.

While many of our assumptions are more than a bit unrealistic, probably the most unrealistic is that the company would declare bankruptcy the moment the value of its assets declined to the par value of its liabilities (i.e., par value of its bonds). If that were the case, the typical recovery for bond holders following defaults would be 100%. In fact, even for senior secured bond holders it averages only about 50%. Thus, companies generally do not declare bankruptcy until the accounting book value of equity gets well below zero (even though the economic book value of equity will never go below zero).

Thus, one must estimate at what point the company really would declare bankruptcy. The value of the assets at which they really would declare bankruptcy is called the "default barrier." The distance from the current value of the assets to the default barrier is called the “distance to default” and is most often measured in terms of standard deviations (vols).



Now suppose that you extract the probability that a company will default on its bonds from the bond price, and you extract the probability the company will default on its bonds from the stock price, as we did above. Now suppose that the bond price implies that the probability of default over the next year is 0.85% and the stock price implies that the probability of default is 0.35%. If markets were perfectly efficient (and if the underlying assumptions were completely realistic), the two implied probabilities of default should agree. But they do not. The higher probability of default implied by the corporation’s bond price suggests that the bonds are “cheap” and the stock is “rich.” So, the hedge fund manager would buy the bonds and short the stock.

Now one might ask, "what has any of this got to do with the derivatives that this book is about?" The answer is in how we choose to execute the strategy. We can substitute credit default swaps for cash bonds, and we can substitute, equity options for cash stock. Let's close this subject by exploring the logic of this just a bit.

If the bond price implies a relatively high probability of default, and if the CDS on the bond reflects a similar relatively high probability of default (relative to what is suggested by the stock price), we could simply enter into appropriate credit default swaps as "credit protection seller." However, because of the possibility that the CDS might not be efficiently priced, relative to the cash bonds, what we would really do is extract the probability of default on the bonds from the quoted spread on the CDS.

Now consider the stock. The volatility of the company's assets, as extracted by treating the stock as a call option on the company's assets, also implies the volatility of the company's stock. To see this, consider our earlier example in which we derived a volatility for the assets of 15. Now notice that there were \$45 of assets for each \$15 of equity. This means that the equity multiplier is 3. (The equity multiplier is one of the leverage ratios and is defined as the ratio of the asset value to the book value of equity.) This implies that the volatility of the stock is 45. One would expect that options on the company's stock would have the same implied volatility (again if all our assumptions were realistic). But that will not necessarily be the case because the options might be mispriced relative to the stock.

For this reason, we can reverse the procedure. That is, look at at-the-money options on the stock, back out the implied volatility of the stock. Then, by dividing by the equity multiplier, we can infer the implied volatility of the assets. From the inferred volatility of the assets, we can obtain the probability of default on the bonds.

Suppose then that the spread on one-year CDS implies a probability of default that is higher than the probability of default implied by one-year at-the-money equity options. Then we would want to use CDS to "sell credit protection" and use equity options to "sell volatility." Thus, we are playing both legs in a capital structure arbitrage with derivatives rather than with cash instruments.

If this logic is correct, then one would expect to find a high degree of correlation between the spread associated with the CDS on a company's bonds and the implied volatility associated with options on the company's stock, provided that they both have the same tenor. Repeated studies have indeed verified this to be the case.

There are many other examples we could offer of the use of derivatives in the hedge fund search for alpha opportunities, but the two provided in this chapter should be sufficient to make the case that many strategies employed by today's hedge fund managers would not be possible in the absence of derivatives and the mathematical logic that drives their pricing.

Bibliographic Information

Merton, Robert C. 1974. "On the pricing of corporate debt: The risk structure of interest rates," *Journal of Finance*.